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Robust Multi-Objective Hybrid Flow Shop Scheduling

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ABSTRACT

Scheduling is an important decision-making process that aims to allocate limited resources to the jobs in a production process. Among scheduling problems, Hybrid Flow Shop (HFS) scheduling has good adaptability with most real world applications including innumerable cases of uncertainty of parameters that would influence jobs processing when the schedule is executed. Thus a suitable scheduling model should take parameters uncertainty into account. The present study develops a multi-objective Robust Mixed-Integer Linear Programming (RMILP) model to accommodate the problem with the real-world conditions in which due date and processing time are assumed uncertain. The developed model is able to assign a set of jobs to available machines in order to obtain the best trade-off between two objectives including total tardiness and makespan under uncertain parameters. Fuzzy Goal Programming (FGP) is applied to solve this multi objective problem. Finally, to study and validate the efficiency of the developed RMILP model, some instances of different size are generated and solved using CPLEX solver of GAMS software under different uncertainty levels. Experimental results show that the developed model can find a solution to show the least modifications against uncertainty in processing time and due date in an HFS problem.

1. Introduction

Scheduling is an important task in production systems that aims to optimize one or more objectives under activity constraints and limited resources [1]. Its applications are ranging from production and manufacturing to transportation and logistics systems [2]. In fact, the purpose of production scheduling is to use various kinds of resources in an optimal way in a time based schedule. Development of an efficient scheduling method can results in productivity improvement of an organization [3]. One of the scheduling environment that is adaptable with most real world industry problems is the Hybrid Flow Shop (HFS) which is a difficult problem to solve [4]. A set of jobs flow through multiple stages in the

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same machine order in a HFS problem. There are several parallel machines at each stage, which can be identical, uniform and unrelated. Jobs have to be processed by one of the machines at each stage, and the flow of jobs through the shop is unidirectional [5] and [6]. There are several research dealt with the HFS with identical machines [7]-[9]. Besides, the main objective functions of the reviewed HFS models are the maximum value, the average, the sum or the weighted sum of the average, the sum or the weighted sum of completion time of jobs [10]-[13], delays or tardiness of jobs [4] and [9], and flow time [14] and [15].

To model the HFS problem, there are mainly six methods. Two methods are position-based approach, three methods are based on the precedence relation between operations and one is based on discrete time periods. The position-based methods assume that each machine (Wanger's modelling [16]) or each stage (Naderi's modelling (i) [12]) has several sections regarded as a processing position according to the time sequence. A schedule plan should arrange jobs at each processing position. Precedence relation-based methods consider: i) precedence relation between two adjacent or non-adjacent operations on the same machine (Manne's modelling [17]), ii) only two adjacent operations (Guinet et al. [18]) on the same machine, and iii) precedence relation of all jobs at each stage (Naderi's modelling (ii) [12]). The method based on the discrete periods (Bowman's modelling) introduces binary decision variables. These variables determine whether an operation processes on one machine in a certain period. Obviously, the large number of binary decision variables makes solving the problem harder [19]. Recently, Meng et al. [20] further have studied the above mentioned models for the HFS problem where their objectives were to minimize the makespan. Based on the experimental results, they have shown that all six existing models are correct and only are formulated based on different ideas.

The HFS problem under uncertainty has received much attention in the recent years [9], [10], [13], [15], [21], and [22]. Since real scheduling problems might be affected by various sources of uncertainties, ignoring them may lead to poor schedules [2] and [23]. Therefore, scholars have introduced different methods where uncertainty is directly taken into account. Some methods consider the random variables as input and some of them are worst-case approaches in which uncertain parameters belong to uncertainty sets. These frameworks are called respectively stochastic programming and Robust Optimization (RO). Due to hedging against adverse conditions of a system and being more insensitive against the future fluctuations of parameters, robust schedules are desirable from a practical perspective [2].

In the sequel, first we review some robust optimization related research. Li and Ierapetrito [24] addressed uncertainty in scheduling problem and studied three robust counterpart optimization formulations. They concluded that the most appropriate model to deal with uncertainty is the formulation proposed by Bertsimas and Sim (B&S) [37]. Rahmani and Heydari [14] studied the flow shop scheduling under unexpected arrivals of the new jobs and uncertain processing times. They proposed a new approach in which an initial robust solution was determined proactively against uncertain processing times at first. Then, an efficient reactive method was produced to deal with unexpected disruption. Based on their computational results, the proposed method could produce better solutions in comparison with the classical heuristic approaches. Nagasawa et al. [25] proposed a robust schedule to limit the peak power consumption by considering unexpected fluctuation in the processing time. They performed simulations in order to find the optimal amount of idle time, which leads to having longer makespan and decreasing the production efficiency. They also proposed a robust scheduling model that considered random processing times and the peak power consumption. Based on the results of experiments, the performance of the schedule produced by the proposed method was superior to the initial schedule and to a schedule produced by another method. Thus, they concluded that the use of random processing times could limit the peak power. Shahnaghi et al. [26] provided a robust model for flow shop with batch processing by using B&S robust optimization approach in which the processing times and the size of jobs are uncertain. Based on the PSO algorithm results, B&S approach has better performance compared to the Ben-Tal model. Emami et al. [27] presented a robust optimization approach for the order acceptance scheduling problem with non-identical parallel machines based on the B&S approach. Job profits and the processing times considered random parameters. Hamaz et al. [28] applied the B&S approach to formulate the scheduling problem with uncertain processing times as a two-stage robust optimization problem. Ding et al. [29] studied the problem of program performance scheduling with accepting strategy under uncertainty of actual situation. This paper built a min-max robust optimization model based on B&S approach to minimize the performance cost and determine the sequence of the programs. Jamili [30] intended to deal with a new variant of job shop scheduling problem under uncertain processing times. In this research, a robust model was proposed by applying the B&S approach. Next, some algorithms were employed to solve the practical cases that were demonstrated as a proof of the effectiveness of the new approach. Bougeret et al. [2] studied the scheduling with budgeted uncertainty on a single machine that minimizes the makespan on parallel and unrelated machines. The processing times could take any value in the budgeted uncertainty set introduced by B&S [2]. Goli et al. [31] proposed a novel Robust Mixed-Integer Linear Programming (RMILP) model for the flow shop scheduling problem with outsourcing option based on the B&S approach, which uncertain processing time is one of their assumption. The comparison analysis demonstrated the superiority of the proposed model against the previous non-linear programming model in the literature [31]. *Table 1* also summarizes these researches.

Table 1. *Studies on the applying B&S approach to various scheduling problems.*

year	Refrence	Problem	Uncertain Parameters
2019	*Bougeret et al. [2]*	*Single machine scheduling.*	*Processing time.*
2019	*Goli et al. [31]*	*Flow shop scheduling.*	*Processing time.*
2019	*Jamili [30]*	*Job shop scheduling.*	*Processing time.*
2019	*Hamaz et al. [28]*	*Basic cyclic scheduling*	*Processing time.*
2018	*Ding et al. [29]*	*Program performance scheduling.*	*Actual situation strategy.*
2017	*Emami et al. [27]*	*Order acceptance scheduling.*	*Job profits and the processing times.*
2016	*Shahnaghi et al. [26]*	*Flow shop.*	*Processing times and the size of jobs.*

The present study applies the B&S robust optimization approach to an extended version of the HFS problem of Meng et al. [20] under uncertainty. This extended model considers the identical parallel machines and includes two uncertain parameters: processing time P_{ik} and due date d_i. Besides, we consider the total tardiness of jobs along with the makespan as a second objective function.

The remaining part of this paper is organized as follows: Section 2 briefly discusses B&S robust optimization approach. In Section 3, the HFS model and its robust counterpart formulation are discussed. Since the proposed problem is a multi-objective one, Section 4 introduces the applied fuzzy goal programming approach for solving the MILP model. Section 5 presents the computational results, model validation and sensitivity analysis. Finally, Section 6 describes the conclusions and future research directions.

2. B&S Robust Optimization Approach

One of the widely used and reliable approaches for dealing with uncertainty is the so-called RO. It has been applied in different real world applications, see for example [31]- [33]. This approach is capable to obtain a solution close to the optimal solution that is guaranteed to be feasible and good for all or most possible realizations of the uncertain parameters [34]. Applying RO to scheduling planning cause to generate more reliable schedule to minimize the effect of disruptions when the schedule is implemented [10]. Soyster [35] was the first who formulated robust linear optimization, but his model was too conservative [35]. Ben-Tal and Nemirovski [36] have introduced an efficient algorithm for modeling uncertainty of data based on ellipsoidal uncertainty sets. Being a conic quadratic problem, this formulation cannot be suitable for optimizing discrete problems. Bertsimas and Sim [37] have developed a different robust formulation that is linear, capable to control the level of conservatism and readily extends to discrete optimization problems. Based on the robust formulation of [37], we propose a robust HFS model under uncertain processing time and due date. In the sequel, first we review Bertsimas and Sim approach.

Consider the linear optimization problem

$$\begin{aligned}
&\min_{x} \sum_{j\in J} \widetilde{a_{0j}} x_j, \\
&\text{s.t.} \\
&\sum_{j\in J} \widetilde{a_{ij}} x_j \le b_i \qquad \forall\, i \in I, \\
&l_j \le x_j \le u_j \qquad \forall\, j \in J.
\end{aligned} \tag{1}$$

Let J denotes the set of coefficients $a_{ij},\ j \in J$ that are associated with parameter uncertainty, i.e. $j \in J$ takes values according to a symmetric distribution with mean equal to the nominal value $\overline{a_{ij}}$ in interval$[\,\overline{a_{ij}} - \widehat{a_{ij}}\ ,\overline{a_{ij}} + \widehat{a_{ij}}\,]$. The $\widehat{a_{ij}}$ is known as perturbation of uncertain parameter. A parameter Γ that takes values in the interval $[0, |J|]$ is introduced to control the level of conservatism of the solution. The main idea is that only a subset of the coefficients $a_{ij},\ j \in J$ will usually change. Therefore, being protected against the fluctuations, B&S approach considers that $\lfloor\Gamma\rfloor$ of these coefficients are allowed to change, and one of the coefficient a_{it} changes by$(\,\Gamma - \lfloor\Gamma\rfloor)\widehat{a_{it}}$. Based on this approach, the robust solution will be feasible if less than Γ uncertain coefficients change. Furthermore, there is high probability that the obtained robust solution will be feasible even if more than Γ change [37]. Based on this, the proposed Robust Linear Programming is developed as follows:

$$\begin{aligned}
&\min t, \\
&\sum_{j\in J} \overline{a_{0j}} x_j + \max_{\{S\cup\{t\}|S\subseteq J,|S|=\lfloor\Gamma\rfloor\ ,t\in S\backslash J\}} \left\{ \sum_{j\in S} \widehat{a_{0j}} |x_j| + (\,\Gamma - \lfloor\Gamma\rfloor)\widehat{a_{0t}}|x_t| \right\} \le t, \\
&\sum_{j\in J} \overline{a_{ij}} x_j + \max_{\{S\cup\{t\}|S\subseteq J,|S|=\lfloor\Gamma\rfloor\ ,t\in S\backslash J\}} \left\{ \sum_{j\in S} \widehat{a_{ij}} |x_j| + (\,\Gamma - \lfloor\Gamma\rfloor)\widehat{a_{it}}|x_t| \right\} \le b_i\ \forall\, i \in I.
\end{aligned} \tag{2}$$

Based on the *Model (2)*, if Γ is an integer, the constraint is protected by $\beta(x,\Gamma) = \max_{\{S\cup\{t\}|S\subseteq J,|S|=\lfloor\Gamma\rfloor\ ,t\in S\backslash J\}} \{\sum_{j\in S} \widehat{a_{ij}}|x_j|\}$; when $\Gamma = 0$, the constraints turn to that of the nominal problem; if $\Gamma = |J|$, the *Model (2)* turns to Soyster's method because it means that all of the coefficient a_{ij} will change. Hence, the robustness of the approach against the level of solution conservatism can be adjusted by varying $\Gamma \in [0, |J|]$. *Model (2)* in the current form is nonlinear. To reformulate it as a linear model, the conservation function of i^{th} constraint is calculated for the given vector x^*:

$$\beta(x^*.\Gamma) = \max_{\{S\cup\{t\}|S\subseteq J,|S|=\lfloor\Gamma\rfloor,t\in S\backslash J\}} \left\{\sum_{j\in S} \widehat{a_{ij}}|x^*{}_j| + (\Gamma - \lfloor\Gamma\rfloor)\widehat{a_{it}}|x^*{}_t|\right\} \le b_i, \tag{3}$$

where it is equal to the objective function of the following linear optimization problem:

$$\begin{aligned} &\beta(x^*,\Gamma) = \max \sum_{j\in J} \widehat{a_{ij}}|x^*{}_j|\, R_{ij}, \\ &\sum_{j\in J} R_{ij} \le \Gamma, \\ &0 \le R_{ij} \le 1 \qquad \forall j \in J. \end{aligned} \tag{4}$$

The dual model of the above problem is:

$$\begin{aligned} &\min \sum_{j\in J} \rho_{ij} + \Gamma Z_i, \\ &\rho_{ij} + Z_i \ge \widehat{a_{ij}}|x^*{}_j| && \forall i.j \in J, \\ &\rho_{ij} \ge 0 && \forall j \in J, \\ &Z_i \ge 0 && \forall i. \end{aligned} \tag{5}$$

By strong duality, since *Problem (4)* is feasible and bounded for all $\Gamma \in [0, |J|]$, then the dual *Problem (5)* is also feasible and bounded and their objective values coincide. So we have that $\beta(x^*, \Gamma)$ is equal to the objective function value of *Problem (5)*.

Now *Model (2)* can be reformulated as a linear optimization model as follows [37]:

$$\begin{aligned} &\min t, \\ &\sum_{j\in J} \overline{a_{0j}} x_j + \Gamma.Z_0 + \sum_{j\in J} \rho_{0j} \le t, \\ &\sum_{j\in J} \overline{a_{ij}} x_j + \Gamma.Z_i + \sum_{j\in J} \rho_{ij} \le b_i && \forall i \in I, \\ &Z_i + \rho_{ij} \ge \widehat{a_{ij}} x_j && \forall j \in J.\, i \in I \cup \{0\}, \\ &Z_i.\rho_{ij} \ge 0 && \forall j \in J.\, i \in I \cup \{0\}, \end{aligned} \tag{6}$$

where Γ shows the degree of conservatism.

3. Problem Statement

Consider a set of n jobs $(i=1, 2,\dots,n)$ that have to be processed at S stages in sequence. Each job i consists of a chain of operations. An operation should be processed at stage j $(j=1, 2, \dots, S)$ for which there are m_j $(m_j \geq 1)$ identical parallel machines. At least one stage has at least two parallel machines. All jobs have the same machine sequence to be processed and there is a set of m $(K=1, 2, \dots, m)$ total machines at all stages. Since all parallel machines within each stages are identical, the processing time of an assigned job to a machine does not depend on the specific machine (*Fig. 1*).

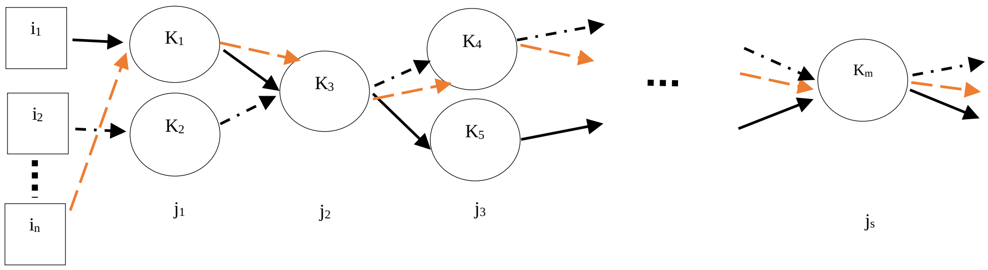

Fig. 1. *Hybrid flow shop scheduling problem.*

Wagner's position-based modelling method [16] is applied in the present paper to develop an HFS model. Hence, we divided each machine into several processing positions according to the time sequence. One position cannot be assigned to more than one job simultaneously. A scheduling plan should arrange the jobs to processing positions of the machines.

To develop the model, *Table 2* and *Table 3* show the assumptions and a description of the notations.

Table 2. *Assumptions.*

Number	Description
1	*At least one stage has at least two parallel machines.*
2	*All the machines have two modes (on and off mode).*
3	*Machines are automatically operated and labor is not included.*
4	*All machines are available at time zero.*
5	*Parallel machines at each stage are unrelated.*
6	*Parallel machines at each stage in terms of capacity, electricity consumption and processing speed, are identical.*
7	*Parallel machines at one stage are independent of other stages.*
8	*Each machine can only process one job at any time and when it is processing a job.*
9	*Machines are always available.*
10	*Breakdown and preventive maintenance are not considered.*
11	*Preparation and movement time between machines are not considered.*
12	*There are j different jobs.*
13	*All jobs are available at time zero and jobs release times are zero.*
14	*Job processing cannot be interrupted, until the completion of job. No interruption once a job has started (non-preemption).*
15	*A job cannot be processed simultaneously on more than one machine.*

Table 3. *A description of notations used in all formulas.*

Notation	Description
i, ii	*Jobs index.*
j, jj	*Stages index.*
k	*Machines index.*
p	*Positions index.*
n	*Number of jobs.*
s	*Number of stages.*
m	*Total number of machines.*
m_j	*Number of parallel machines within stage j, $mj \geq 1$.*
I	*Set of jobs where {1, 2,..., n}.*
J	*Set of stages where {1, 2,..., S}.*
K_j	*Set of parallel machines within stage j where {1,2,...,mj}.*
K	*Set of total machines where {1, 2,...,m}.*
P	*Set of positions of machine k where {1,2,...,n}.*
M	*A very large positive integer.*
p_{ik}	*processing time of job i on machine k.*
d_i	*Due date of job i.*
Z_{jk}	*Parameter that is equal 1 if machine k is at stage j, and 0 otherwise.*
B_{ij}	*Continuous variable that determines the starting time of jth operation of job i.*
S_{kt}	*Continuous variable that determines the starting time of machine k in position t.*
T_i	*Tardiness of job i.*
$Cmax$	*The makespan.*
x_{ik}	*Binary variable that is equal 1 if job i is processed on machine k, and 0 otherwise.*
y_{ikp}	*Binary variable that is equal 1 if job i occupies position p of machine k, and 0 otherwise.*

3.1. MILP Model

We consider the following extended model of Meng et al. [20] that is bi-objective and takes into account uncertainty in processing times and due date:

$$\text{Min } C_{max,} \tag{7}$$

$$\text{Min } \sum_{i} T_{i,} \tag{8}$$

S.t.

$$\sum_{k \in K_j} x_{i.k} = 1 \qquad \forall i.j, \tag{9}$$

$$\sum_{p \in P} y_{i.k.p} = x_{i.k} \qquad \forall i.k \in K, \tag{10}$$

$$\sum_{i} y_{i.k.p} \leq 1 \qquad \forall k \in K.p \in P, \tag{11}$$

$$\sum_{i} y_{i.k.p} \geq \sum_{ii} y_{ii.k.p+1} \qquad \forall k \in K.p \in \{1.2.\dots.n-1\}, \tag{12}$$

$$B_{i.j} + \sum_{k \in K_j} P_{i.k} x_{i.k} \leq B_{i.j+1} \qquad \forall i.1 \leq j \leq s-1, \tag{13}$$

$$S_{k.p} + \sum_{i} P_{i.k} y_{i.k.p} \leq S_{k.p+1} \qquad \forall k \in K.1 \leq p \leq n-1, \tag{14}$$

$$S_{k.p} \leq B_{i.j} + M(1 - y_{i.k.p}) \qquad \forall i.j.k \in K_j.p \in P, \tag{15}$$

$$S_{k.p} \geq B_{i.j} - M(1 - y_{i.k.p}) \qquad \forall i.j.k \in K_j.p \in P, \tag{16}$$

Where, the *Objectives (7)* and *(8)* are to minimize the makespan and total tardiness,

$$C_{max} \geq B_{i.S} + \sum_{k \in K_S} P_{i.k} x_{i.k} \qquad \forall i, \qquad (17)$$

$$B_{i.S} + \sum_{k \in K_S} P_{i.k} x_{i.k} - d_i \leq T_i \qquad \forall i, \qquad (18)$$

$$T_i . B_{i.j} . S_{k.p} \geq 0, \qquad (19)$$

$$x_{i.k} . y_{i.k.p} \in \{0.1\}. \qquad (20)$$

respectively. *Constraint (9)* determine that each job cannot be assigned to more than one machine at each stage. The relation between the two decisions variables are defined by *Constraint (10)*. *Constraint (11)* represent that any position of a machine processes exactly one job at a time. *Constraint (12)* guarantee that jobs assignment to the machine positions are in sequential order. That is, each position of machines can only be occupied when its previous position of the same machine is occupied. *Constraint (13)* are associated with the operation precedence relations. They guarantee that each operation of each job can be started only if its precedent operation has been completely finished at the previous stage. Based on the *Constraint (14)*, each machine has to process at most one operation at a time. That is, for two adjacent operations assigned to the same machine, the succeeding operation can be only started when the precedent operation has been completely finished. *Constraints (15)* and *(16)* guarantee that the starting time for a position of a machine equals to the starting time of the operation arranged at this position. *Constraint (17)* calculate the makespan and *Constraint (18)* calculate the tardiness of each job. *Constraints (19)–(20)* also define the decision variables.

3.2. Robust Formulation

Suppose that *Model (2)* has two uncertain parameters $\widetilde{p_{ik}} \in [\overline{P_{ik}} - \widehat{P_{ik}} . \overline{P_{ik}} + \widehat{P_{ik}}]$ and $\tilde{d_i} \in [\overline{d_i} - \hat{d_i} . \overline{d_i} + \hat{d_i}]$ where $\overline{P_{ik}}$ and $\overline{d_i}$ are the nominal and $\widehat{P_{ik}}$ and $\widehat{d_i}$ are the perturbation value of these parameters. These parameters cause uncertainty in the *Constraints (13), (14), (17)* and *(18)*. *Constraints (13)* and *(17)* contain $\sum_{k \in K_j} \tilde{P}_{i.k} x_{i.k}$. We consider parameter $\Gamma \in [0.m_j]$ to adjust robustness of the model against the level of conservatism of the solution. The uncertain coefficients set is also K_j ($m_j = |K_j|$). Furthermore, *Constraints (18)* calculate the tardiness of each job *i* and contains $\tilde{d}_i$. Here also, we set parameter $\Gamma'' \in [0, 1]$. Hence, the followings are the robust counterparts of the *Constraints (13), (17)* and *(18)* (based on *Model (6)*):

$$B_{ij} + \sum_{k \in K_j} \overline{P_{ik}} x_{ik} + \sum_{k \in K_j} \rho_{ijk} + \Gamma Z_{ij} \leq B_{ij+1} \qquad \forall\, i.\, 1 \leq j \leq s-1, \qquad (21)$$

$$\rho_{ijk} + Z_{ij} \geq \widehat{P_{ik}} x_{ik} \qquad \forall\, i.\, 1 \leq j \leq s.k \in K_j, \qquad (22)$$

$$\rho_{ijk} . Z_{ij} \geq 0 \qquad \forall\, i.\, 1 \leq j \leq s.k \in K_j, \qquad (23$$

$$B_{is} + \sum_{k \in K_s} \overline{P_{ik}} x_{ik} + \sum_{k \in K_s} \rho_{isk} + \Gamma Z_{is} \leq Cmax \qquad \forall i, \qquad (24)$$

$$B_{is} + \sum_{k \in K_s} \overline{P_{ik}} x_{ik} + \sum_{k \in K_s} \rho_{isk} + \Gamma Z_{is} - (\overline{d_i} - \Gamma'' \hat{d_i}) \leq T_i \qquad \forall\, i. \qquad (25)$$

Constraint (14) contain $\sum_i P_{i.k} y_{i.k.p}$ and parameter $\Gamma' \in [0.n]$. The uncertain coefficients set is I ($n = |I|$). In fact, by assigning the value to $\Gamma . \Gamma'$and Γ'' in all the robust counterparts, a trade-off occurs between the robustness and optimality of the problem. These parameters are determined by the decision maker and their values are dependent on the risk aversion and the importance of the constraint for the decision maker [32, 33]. Based on *Model (6)* that demonstrates the robust counterpart of a constraint (resulted of B&S approach), *Constraints (26)-(28)* enter the problem model instead of the *Constraint (14)* as follows:

$$S_{kp} + \sum_i \overline{P_{ik}} y_{ikp} + \sum_i \rho'_{ikp} + \Gamma' Z'_{kp} \leq S_{kp+1} \qquad \forall\, k.\, 1 \leq p \leq n-1, \tag{26}$$

$$\rho'_{ijk} + Z'_{kp} \geq \widehat{P_{ik}} y_{ikp} \qquad \forall\, i.\, j.\, 1 \leq p \leq n-1, \tag{27}$$

$$\rho'_{ijk}.\, Z'_{kp} \geq 0 \qquad \forall\, i.\, j.\, 1 \leq p \leq n-1. \tag{28}$$

If the value of parameters Γ, Γ' and Γ'' equal to 0, the robust problem reduces to the nominal problem.

4. Multi Objective Optimization by Fuzzy Goal Programming

One of the methods that is widely used in the multi-objective problems is goal programming that presents the efficient solution set. However, selection of the satisfactory solution among this set is not easy for the decision makers. Therefore, the Fuzzy Goal Programming (FGP) introduced by Narasimhan [38] applied by many researchers to solve multi-objective problems [39]-[43].

Consider the following multi-objective model:

$$\text{Min}\ \left(f_1(x).\, f_2(x).\, \ldots.\, f_n(x) \right) \\ x \in X. \tag{29}$$

Based on the idea of FGP, each objective $f_i(x)$ has an associated fuzzy goal which deviations of all goals should be minimized. The membership functions of each fuzzy goal represent the grade of membership of a goal in the fuzzy subset and FGP approach aims to minimize the deviational variables to achieve the highest degree of each membership goals and also the most satisfactory solution [39].

The membership function of each goal is defined based on the Positive-Ideal Solution (PIS) and Negative-Ideal Solution (NIS), which respectively are the best possible value and the feasible worst solution of each objective function. Therefore, the satisfactory degree of 1 is assigned to the PIS as the most preferred value and NIS has the satisfactory degree of 0. Consider m_i as the minimum value and M_i as the maximum value of objective $f_i(x)$. Since all objective functions of the developed MILP model in the present study are of minimization-type, m_i is the PIS for each $f_i(x)$ and has the satisfactory degree of 1 and M_i is the NIS for each $f_i(x)$ and has the satisfactory degree of 0.

It should be noted that if we define the membership functions of the deviations of the goals instead of the goals, the minimax FGP can be applied. Thereby, the objective function of FGP is minimization of the maximum of deviations. Based on this method, the difference between PIS (m_i) and NIS (M_i) calculates the acceptable deviation from the goal. Then, the membership function can be shown as *Fig. 2* and presented by

$$\mu(x)_{|f_i} = \begin{cases} 0 & f_i(x) \geq M_i \\ \dfrac{f_i(x)-m_i}{M_i - m_i} & x \in X \quad . \\ 1 & f_i(x) \leq m_i \end{cases} \tag{30}$$

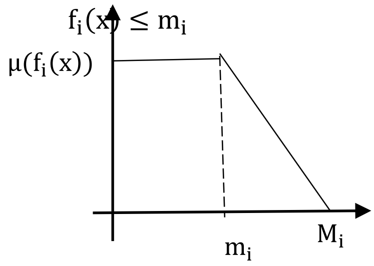

Fig. 2. *Membership function of deviations.*

Therefore, the multi-objective *Model (29)* can be defined as a single objective problem as follows:

$$\min z = \max\{d_i(x) : i = 1 : n\},$$

$$d_i(x) = \frac{f_i(x) - m_i}{M_i - m_i} \ ; \ i = 1.2. \dots . n. \tag{31}$$

$$x \in X$$

It can be clearly seen that if $f_i(x) \to m_i$, then $d_i(x) \to 0$. The above model can be replaced by *Model (32)* which is linear:

$$min \ Z,$$

$$Z \geq \frac{f_i(x) - m_i}{M_i - m_i} \ ; \ i = 1.2. \dots . n. \tag{32}$$

$$x \in X$$

5. Computational Results

To evaluate the efficiency of the developed model, some instances are generated. Initially, we solved the problem with the nominal values. Then, $\Gamma \in [0, m_j]$, $\Gamma' \in [0, n]$, and $\Gamma'' \in [0, 1]$ are determined based on B&S approach. Finally, the perturbation value of uncertain parameter, i.e., the processing time and due date are defined as $\widehat{P_{ik}} = \alpha \overline{P_{ik}}$ and $\widehat{d_i} = \beta \overline{d_i}$ (α and β are predetermined).

5.1. Design of Experiments

The data required to create the problems are shown in *Table 4*. The characteristic of each problem is introduced by A-B-C-D, which shows, respectively, the number of jobs, the number of stages, the number of total machines and the number of machine positions.

Table 4. *Problems with nominal parameters values given to GAMS.*

Number of Instance	Problem Characteristic	Number of Machines per Stages	Nominal Processing Times	Nominal Due Dates
1	*5-2-3-5*	*2* *1*	*[5-10]* *[4-6]*	*U[10-30]*
2	*7-2-3-7*	*2* *1*	*[5-10]* *[4-6]*	*U[20-50]*
3	*8-3-5-8*	*2* *1* *2*	*[20-70]* *[20-60]* *[10-40]*	*U[100-500]*

To solve both the deterministic and robust models of problems given in *Table 3*, CPLEX solver of GAMS software is used in a Laptop with Intel Core i7- 2.6 GHz processor and 8 GB RAM. Moreover, the maximum runtime of the solver is set to 2000 seconds. *Table 5* shows the objectives and their deviations that have been obtained after solving the final developed model by applying FGP approach. The results are presented for the perturbation $\alpha = 0.2$ and $\beta = 0.1$(Robust1) and $\alpha = 0.3$ and $\beta = 0.2$ (Robust2). Moreover, we set Γ=1.5, $\Gamma' = 3$ and $\Gamma'' = 0.7$.

Table 5. *Computational results.*

Instance	Objectives	Deterministic	Robust 1 $(\alpha = 0.2\ \beta = 0.1)$	Robust 2 $(\alpha = 0.3\ \beta = 0.2)$
1	*Cmax*	*25*	*27.1*	*29.7*
	Tardiness	*11.277*	*28.186*	*33.37*
	Devation of Cmax	*0.09*	*0.108*	*0.201*
	Devation of Tardiness	*0.09*	*0.108*	*0.201*
2	*Cmax*	*39*	*42.127*	*48.605*
	Tardiness	*11.2*	*18.683*	*47.610*
	Devation of Cmax	*0.154*	*0.162*	*0.194*
	Devation of Tardiness	*0.132*	*0.111*	*0.194*
3	*Cmax*	*347.036*	*409.226*	*442*
	Tardiness	*161.680*	*255.973*	*434.477*
	Devation of Cmax	*0.469*	*0.102*	*0.2*
	Devation of Tardiness	*0.469*	*0.102*	*0.05*

Figs. 3 (a)-3(c) also plot the objective values of the problems with these three levels of perturbations. It can be found that increasing the perturbation of uncertain parameters not only lead to increasing the objectives, but also has a stronger effect on Tardiness compared to C_max. In fact, these figures illustrate the sensitivity analysis of objectives to level of perturbation of the uncertain parameters.

To compare the impact of uncertainty on scheduling plan, *Figs. 4* and *5* show the Gantt charts of the obtained solutions for *Problem 2* (7-2-3-7) with and without uncertainty of parameters. As can be seen, uncertainty not only leads to greater maximum of completion times (it is 42.5 when we do not consider the uncertainty and 54.6 when the uncertainty is considered), but also changes the assignment of jobs to machines and their sequential order. For example, at stage 1, job 5 is assigned to machine k_1 without and to machine k_2 with uncertainty. Moreover, this job is assigned to 2nd machine position without and 3rd machine position with uncertainty. It should be noted that the developed robust counterpart formulation in this paper is capable to guarantee the feasibility. Hence, a schedule plan is created so that the uncertainty of data leads to the least possible modifications.

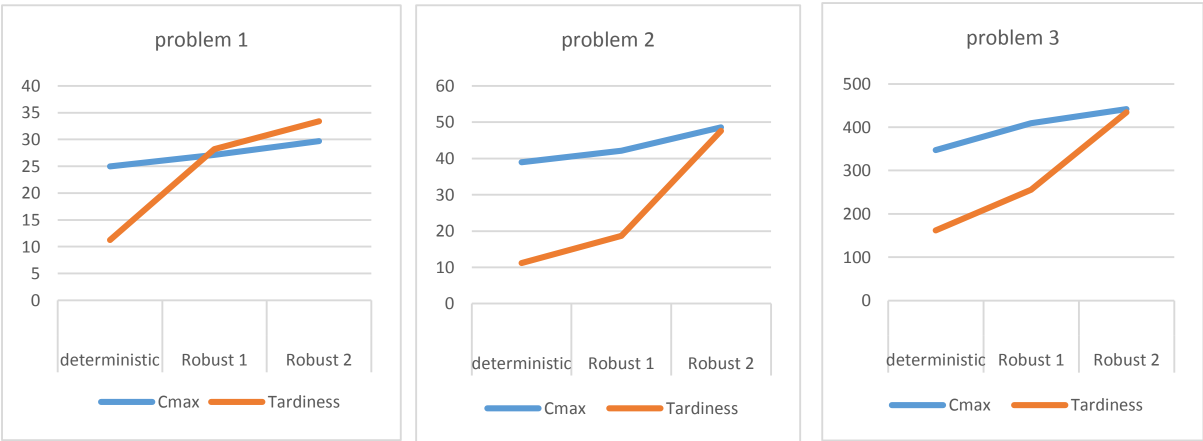

Fig. 3. *The effect of perturbation of uncertain parameters on objective functions.*

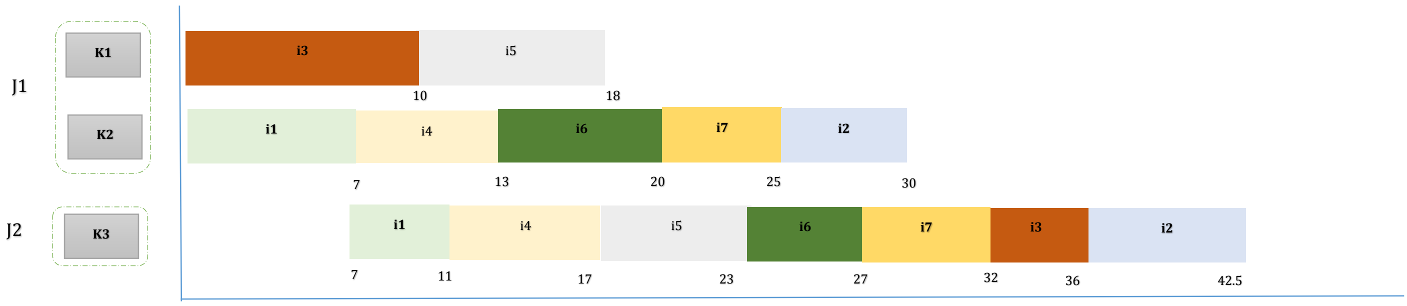

Fig. 4. *The robust schedule of problem 2 without uncertainty of parameters.*

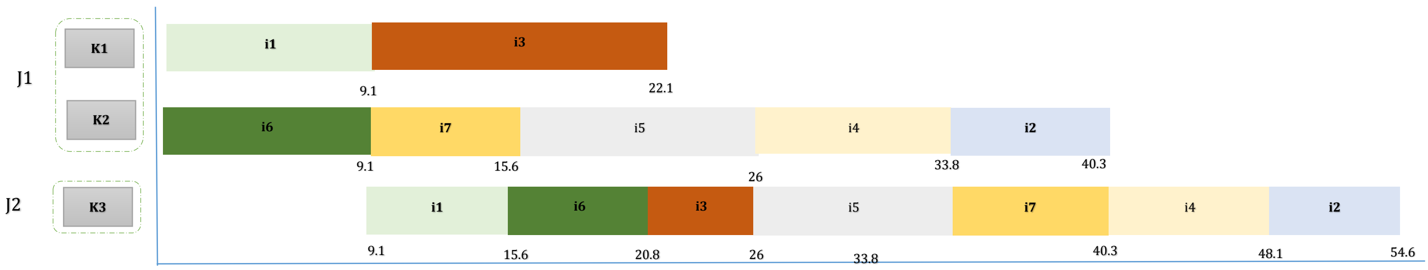

Fig. 5. *The robust schedule of problem 2 with uncertainty of parameters.*

Besides the above experiments, the robust counterpart problem is solved for the different degrees of conservatism. The effect of changing the protection level on the objective function values for the problem 2 with $\Gamma \in [0.2]$, $\Gamma' \in [0.7]$ *and* $\Gamma'' \in [0.1]$ is illustrated in *Fig. 6*. As it can be found, tardiness objective function is more affected by changing the conservatism level. Hence, according to *Fig. 6 (b)* compared to the other charts, when protection level increases, the objective values increase more. That is, the more jobs (n) process with uncertain time, the larger the completion time (C_max) and the larger delay in due date (Tardiness). Moreover, Γ'' is associated with due date based on which the tardiness objective function is claculated. So, different levels of this parameter strongly affects the tardiness compared to the C_max as shown in *Fig. 6 (c)*.

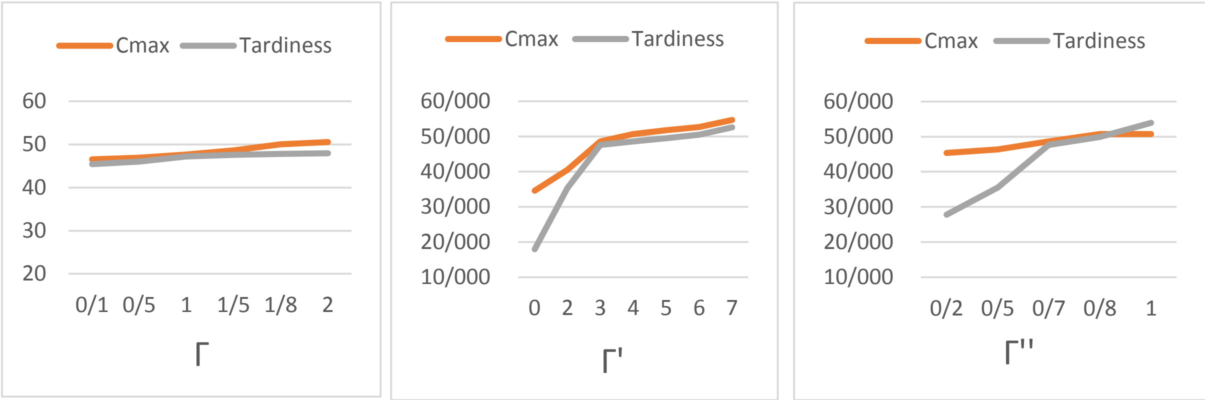

Fig. 6. *Objective values as a function of conservatism levels for Problem 2.*

For further sensitivity analysis and to have better understanding of the objective functions behavior, we compare the effect of robust optimization on the objective values of *Problem 2* (7-2-3-7). The deterministic and robust models with 20% perturbation for all uncertain parameters ($\alpha = \beta = 0.2$) are solved. *Table 6* shows the obtained objective values and deviations for three states: without any protection ($\Gamma = \Gamma' = \Gamma'' = 0$), with the maximum protection ($\Gamma=2$, $\Gamma' = 7$ and $\Gamma'' = 1$) and with the predetermined protection ($\Gamma=1.5$, $\Gamma' = 3$ and $\Gamma'' = 0.7$). In the absence of protection of the processing time and due date, the values of C_max and Tardiness are 39 and 11.2. However, with maximum protection that turns to Soyster's [35] method, the objective values is increased. Compared to the maximum protection, when we reduce the conservatism levels to a predetermined protection mentioned above, the objective values change and become better. In fact, reducing the conservatism level leads to improve the objective functions. As mentioned before, the FGP approach that is applied to solve this two-objective problem tends to minimize the deviations of ideal solution. Furthermore, we have three parameters as conservatism level with different range values. Therefore, it is reasonable that reducing all the conservatism levels simultaneously may affect the objective values in various behaviors. However, we can be sure that this final solution has the minimum deviation from the ideal solution for all objectives.

Based on the above discussion, it is clear that changing the protection level effects the objective values. An important observation is that B&S formulation succeeds in reducing the price of robustness, that is, we do not heavily penalize the objective function value in order to protect ourselves against constraint violation.

Table 6. *Objective values for different degrees of conservatism.*

Problem	**Objectives**	**Deterministic**	**Predetermined Protection**	**Maximum Protection**
7-2-3-7	*Cmax*	*39*	*39.6*	*43.2*
	Tardiness	*11.2*	*16.7*	*27.335*

6. Conclusion

One of the complicated scheduling problems that has good adaptability with real production systems is the HFS environment. This paper suggested a multi-stage HFS model under uncertain processing time and due date with objectives aim to minimize the total tardiness and makespan. The robust optimization based on Bertsiams and Sim approach was applied in order to deal with the uncertainty and Fuzzy Goal Programming is also implemented to solve this multi-objective problem. Based on the computational results of solving three different-sized instance problems, this model was capable to find the satisfactory sequence of jobs for each machine at each stage so that the uncertainty of data leads to the least possible modifications. However, uncertainty changed the assignment of jobs to machines and their sequential order. Moreover, the larger the level of perturbation was, the larger objective function values of the solutions were. Increasing the degree of conservatism also made the objectives to become worse and effected the tardiness stronger than makespan. Besides, the total tardiness of jobs is more affected by changing the protection level of the due date than the processing time. It should be noted that the CPLEX solver of GAMS could solve the extended RMILP model of HFS for the small and medium size problems. It could obtain the satisfactory solution within the time limit and raise the lower bounds in order to prove the optimality of the solution.

One may consider the followings as the future research directions:

- Adding further conditions to the RMILP model such as sequence dependent setup time, precedence constraints between operations from different jobs, preemptions, machine break down, no-wait jobs, new job arrivals.
- Extending the developed model to uniform and unrelated parallel machines.

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