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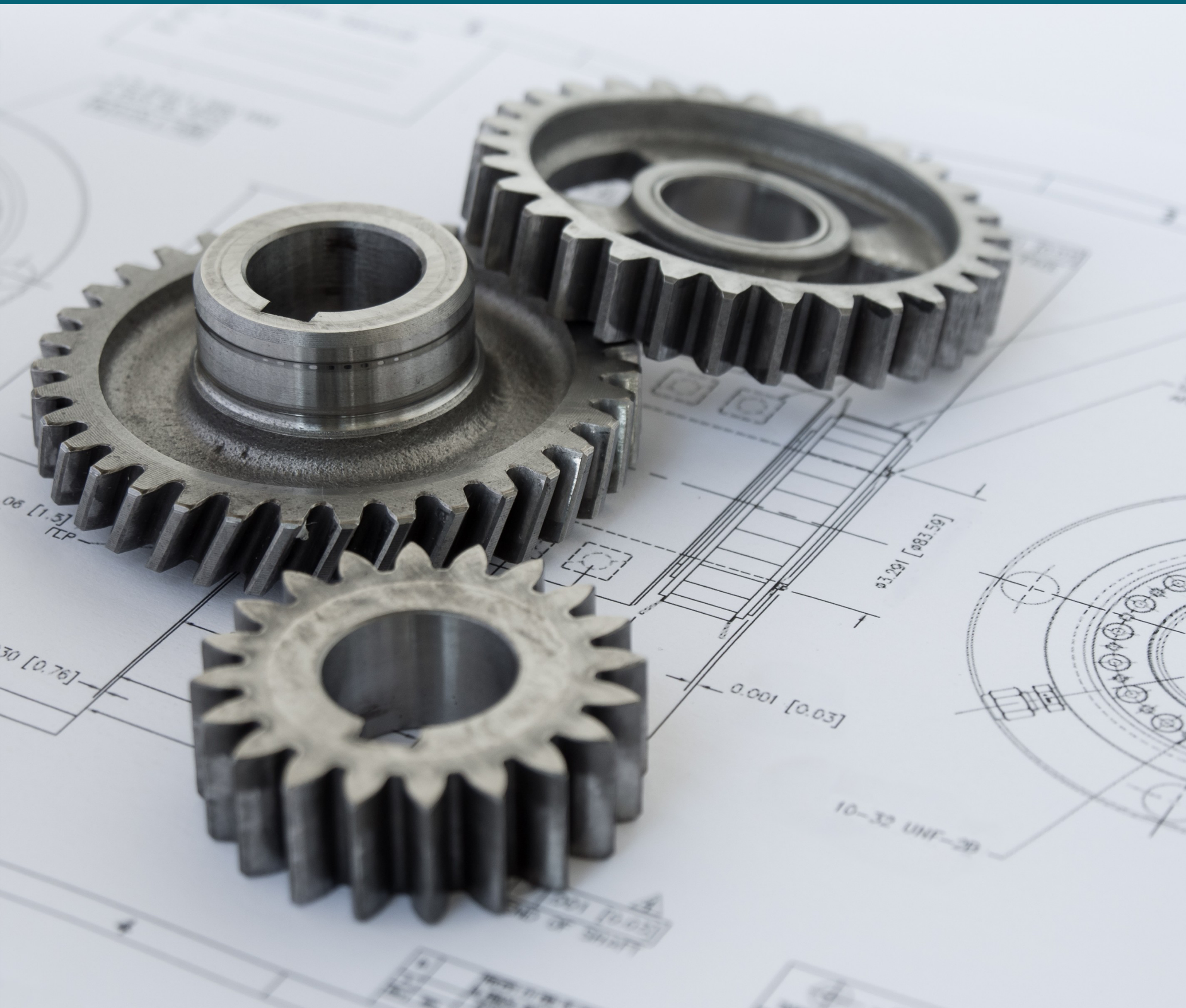

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Analysis of TQM Implementations in the Floor Tiles Industry

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Abstract

The floor tiles industry in accordance with state regulations is one of the manufacturing industries that can participate in Indonesia into a developed industrial country, supported by the availability of various technologies and product innovations so that domestic floor tiles can be accepted. In the face of increasing competition, domestic floor tiles industry trying to improve the quality of the products by implementing a strict standardization system for production processes and products produced by implementing Total Quality Management (TQM). Based on the components that influence of TQM, variables of leadership, technical competence and organizational culture are closely related to certain factors. A leadership pattern that focuses on goals or targets determined and taken based on scientific thinking by taking into account the various parties involved will improve the quality. Education and training as well as employee involvement by providing controlled independence as well as improving the technical quality.

Keywords: Leadership, Technical competence, Organizational culture, TQM, Floor tiles.

1 | Introduction

The Indonesia is one of the biggest manufacturing industry bases in Southeast Asia with a percentage of 20.27% in the economy on a national scale. The manufacturing industry in Indonesia developed current able to swift the role of base on commodity to manufacturing. In today's highly dynamic global business environment, the customers want the best quality of international competition becomes more intense and it becomes very clear that only organizations committed to providing the best quality are able to continue to grow. Quality is a very relevant concept and is a strategic factor that can leading competition role with the success in organization [1]. Placing a high strain on quality enables organizations to meet the needs and wants of customers accurately, and ultimately leads to the realization of the better business competition [2]. Total Quality Management (TQM) is one of the most commonly adopted and prominent quality improvement philosophies in the development business environment. The floor tiles industry in accordance with state regulations is one of the manufacturing industries that can participate in Indonesia into a developed industrial country, supported by the availability of various technologies and product innovations so that domestic floor

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tiles can be accepted abroad market. Asosiasi Aneka Industri Keramik Indonesia (ASAKI) is optimistic that the prospects for the floor tiles industry are still bright. ASAKI even projects that the consumption of the country's ceramic producers could increase to 70% by the end of 2020, higher than the achievement rate of the floor tiles industry over the past 5 years and the growth of the floor tiles sector is projected to be in the range of 5-6%. Based on the data, ASAKI says that the consumption per capita of floor tiles in Indonesia continues to increase every year, while the production capacity of the floor tiles industry has not increased. Meanwhile, the surge in exports occurred in the United States which reached 130%, followed by the Philippines 60%, and Taiwan 40%. In the face of increasing competition, domestic floor tiles industry trying to improve the quality and productivity of their products by implementing a strict standardization system for production processes and products produced by implementing TQM. Several studies claim the major benefits of influencing TQM are productivity, quality, performance improvement, cost reduction, and overall waste elimination that lead to organizational performance [3]. Some empirical evidence suggests a direct and indirect TQM approach [4]. Several researchers have found important effects of influence TQM [5]. The scientific findings described above, it shows that influence of TQM a positive and significant on the operational performance of an organization. The most effective leadership style for the successful influence of TQM depends on cultural background embedded in the company [6].

2 | Literature Review

TQM is the relationship between quality systems and applications that is closely related to competitiveness and performance in a company [7]. Technical competencies related to TQM include technical aspects that refer to management tools, techniques and practices. High leadership efficiency in implementing the nine principles of TQM effectively is capable of producing better quality products [8]. Competency-based training has a positive correlation with 5S and TQM approach improvement, advantage, R&D and quality performance as intermediaries in the influence of TQM, while competition and industry entry are barriers in TQM [9]–[11]. TQM provides practical practices in management, processes and human resources to improve material services at all processes and levels to meet current and future customer needs [12], [13]. Strategic planning and development in production management is related to process strategy i.e., the approach of the organization to convert resources to goods and services with the aim of creating a process that can produce products that meet customer needs in cost and other management constraints [14]. Meanwhile, leadership style is more likely to impact continuous improvement and is considered the dominant TQM approach rather than innovation [15]. Causes of lack of leadership in the implementation of TQM programs include lack of involvement and commitment of senior managers; the combination of leadership that takes place in the organization; and the influence of foreign political leadership. Managers must be aware of cultural values in organizations as these values influence TQM practices and organizational performance [16]. Types of adhocracy and group culture are the cultures that most support the influence of TQM practices [17]. Organizational culture is the part of factor in improving the adoption of TQM practices [3]. Leadership and competency variables are interrelated in improvement in an effort to promote improved employee and company performance [18]. Leadership has a close relationship with competence, as strong leadership can motivate employees to improve their competencies. TQM has consequences, namely strong leadership as it is the key to the success of strategic plans and achieving quality goals [19]. Transformational leadership has a positive impact of influence TQM [20]. The most widely used TQM approach in the floor tiles industry is multifunctional workers and quality control, while the least used is group technology, reducing preparation time and kanban.

3 | Methods

The areas of research conducted in this research are the areas of management in particular leadership, technical competence, organizational culture, influence of TQM. The study design is designed as operational engineering so that it can be carried out perfectly to minimize the element of error, then in this study also formed a study design that contains the planning and implementation of the study.

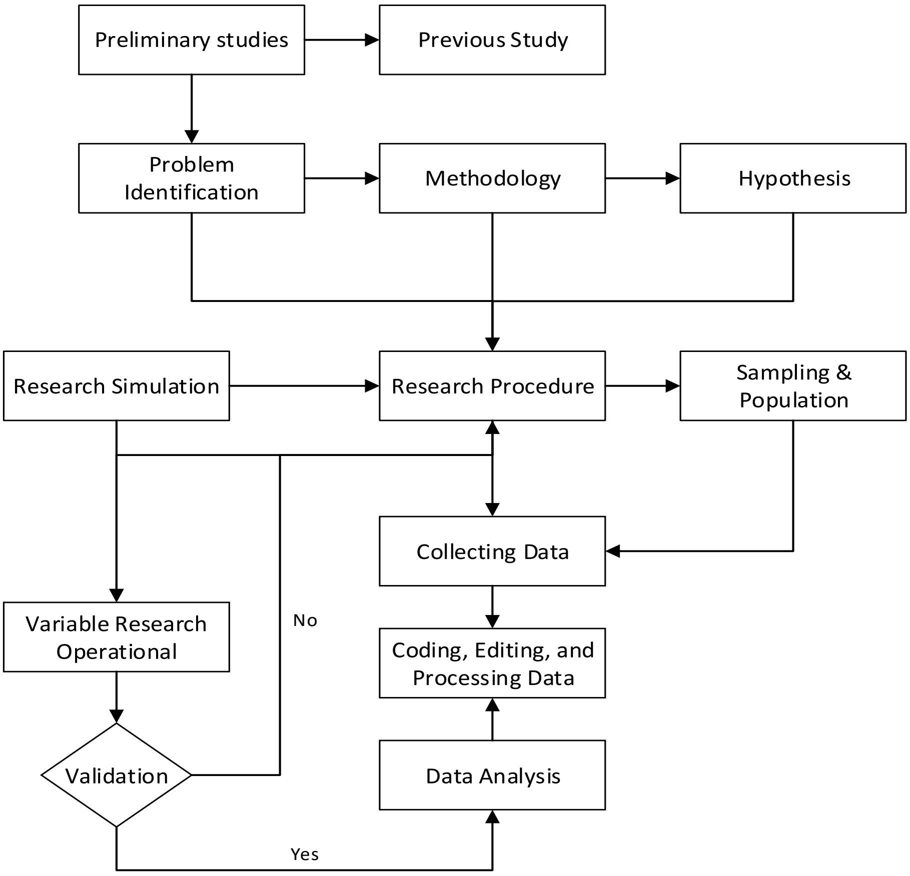

Fig. 1. Research framework.

Research problems need to be answered with reference to references from the literature and previous research, so that a thinking framework is formed that then produces research hypotheses. To develop the thinking framework that has been formed, a research design is prepared that includes: operational variables, data collection instruments used that must be validated. Next, data collection is done through predefined instruments. The subjects of this study are operating workers in the floor tiles industry. In this study, the method used is Structural Equation Modeling (SEM) technique. Some guidelines for determining the sample size for SEM are given as follows [18]:

I. If parameter estimation uses the maximum probability estimation method, the recommended sample size is between 100 and 200, with a minimum sample of 50.
II. As much as 5 to 10 times the number of parameters in the model.
III. Equivalent to 5 to 10 times the number of real variables (indicators) of all latent variables.

The criteria used in the selection of the sample for this research is the number of operating employees in the floor tiles industry, so the number of samples using the Slovin's formula is:

$$n = N/(1 + Nx(e^2)).$$

These techniques with a 95% confidence level, the minimum number of samples obtained is:

$$n = \frac{1140}{1 + 1140 \times 0{,}05^2)} = \frac{1140}{3{,}82} = 298{,}42.$$

Thus, the minimum sample required in this study is 298 operating workers in the floor tiles industry.

4 | Result

This research was conducted on four floor tiles companies in Indonesia with 368 employees. These employees come from a variety of educational backgrounds, years of service and positions. The educational demographics of employees who are the sample of this study are divided into 3 categories,

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namely high school, diploma and bachelor. Based on the observation, it is known that a total of 324 employees or 88.04% of the sample of employees are high school graduates, 16 employees (4.35%) are diploma graduates and 28 employees (7.61%) are bachelor graduates. This shows that most of the operating division workers have no education below secondary school (primary and secondary school) and the average is secondary school education so the operations division workers already have relatively good basic skills and have good work motivation, as well as easier to adapt. The results showed that the majority of the sample of employees involved in this study have a working more than 15 years is 46.74%, employees with 10-15 years is 5.98%, employees with 5-10 years is 11.41%, employees with 1-5 years is 22.55%, and 13.32% of employees with less than 1 year of service. Based on the position, it is known that 74.46% of the respondents are operator; 16.03% of respondents are leaders; 7.07% of respondents were supervisors; 1.36% are superintendent; and 1.09% were managers. Measurement of the validity of the questionnaire was conducted using the Pearson product moment method, which is the result of all questionnaires in the form of correlation scores, if there is a correlation between total score and the score of each question greater than 0.3 then it is valid [14].

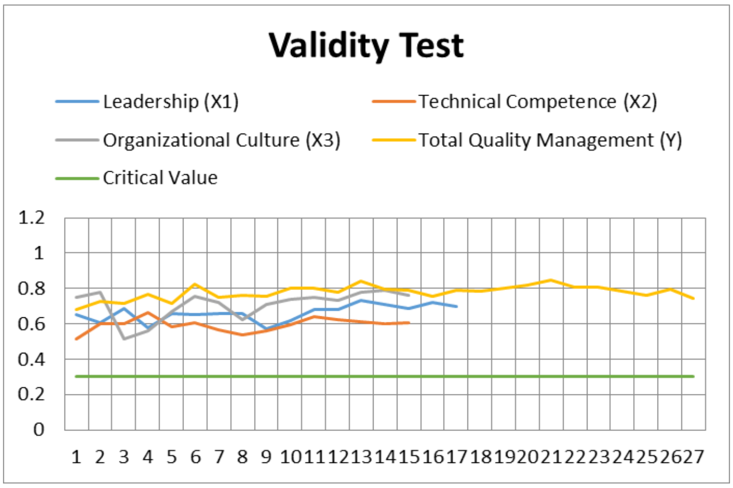

Fig. 2. Validity test.

The reliability of the instruments in this study was estimated using Cronbach's Alpha formula and assisted with the help of SPSS software. The reliability of the instrument is based on validated data that is 89 question items given to 369 respondents showed an average reliability coefficient of 0.979 for variable X_1 of 17 items, average reliability coefficient of 0.899 for variable X_2 as of 15 items, average reliability coefficient of 0.912 for variable X_3 of 15 items, an average reliability coefficient of 0.806 for variable Y of 27 items. The reliability coefficient is quite high. *Table 1* shows the results of the reliability estimates that have been carried out.

Table 1. Results of the reliability estimates.

Variable	Cronbach' Alpha	Critical Value	Information
Leadership	0.979	0.700	Reliable
Technical competence	0.899	0.700	Reliable
Organizational culture	0.912	0.700	Reliable
TQM	0.806	0.700	Reliable

Based on test results with a significance level of 0.05, Asymp values were obtained. Significance for $X_1 = 0.072$; $X_2 = 0.532$; $X_3 = 0.347$; $Y = 0.232$; for each variable is greater than the significance value of 0.05.

Table 2. Results with a significance level of 0.05.

One-Sample Kolmogorov-Smirnov Test					
		X_1	X_2	X_3	Y
N		151	151	151	151
Normal parameter S a,b	Mean	-0.4120	-0.1450	-0.8660	-0.5460
	Std. deviation	4.122 58	6.182 76	3.455 67	4.852 89
Most extreme difference S	Absolute	0.105	0.435	0.176	0.132
	Positive	0.105	0.563	0.799	0.246
	Negative	-0.103	-0.764	-0.235	-0.463
Kolmogorov-smirnov Z		1.288	1.358	1.685	1.246
Asymp. Sig. (2-tailed)		0.072	0.532	0.347	0.232
a. Test distribution is normal.					
b. Calculated from data.					

Variable of leadership is an exogenous variable that is the process of directing and influencing activities that are related to the work of group members, the leader has a task to strive to make the group achieve its goals well in optimal cooperation. The leadership variable (X_1) consists of 4 dimensions namely director, communicator, decision making and motivation. Respondents' response to leadership on average is 4.10, it is known that the leading dimension is 4.10, the communicator dimension is 4.13, the decision making dimension is 4.12 and the motivation dimension is 4.04. Based on these values, the four dimensions of the leadership variable are in the good category.

Table 3. Responses to leadership (X_1).

Dimension	Response						Average	Categories
	1	2	3	4	5	Total		
Leading	1	5	50	215	97	368	4.1	Good
Communicator	1	4	48	210	105	368	4.13	Good
Decision making	0	5	43	224	96	368	4.12	Good
Motivator	0	5	57	224	82	368	4.04	Good
Total score							16.39	
Average							4.10	
Standard deviation							0.03	
Range							4.07-4.13	
Categories							Good	

Next, respondents' response regarding technical competence has an average value of 4.20 in in the very good category, it is known that the knowledge dimension 4.27 is in the very good category, the expertise dimension 4.04 is in the good category, and the attitude dimension 4.29 is in the very good category.

Table 4. Responses to technical competence (X_2).

Dimension	Response						Average	Categories
	1	2	3	4	5	Total		
Knowledge	1	1	25	210	131	368	4.27	Very good
Expertise	0	3	59	234	72	368	4.04	Good
Attitude	0	1	31	208	128	368	4.29	Very good
Total score							12.6	
Average							4.20	
Standard deviation							0.113	
Range							4.09-4,31	
Categories							Very good	

Next, respondents' response regarding organizational culture has an average value of 3.98 in the good category, known innovation dimension 3.81 included in the good category, outcome orientation dimension 4.11 included in the good category, aggressive dimension. Attitude is 4.07 referenced in good category, stability dimension 3.93 is in good category, and attention dimension to item 3.96 is in good category.

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Table 5. Responses to organizational culture (X_3).

Dimension	Response						Average	Categories
	1	2	3	4	5	Total		
Innovation	4	20	79	206	59	368	3.81	Good
Outcome orientation	1	3	52	211	101	368	4.11	Good
Aggressive	2	3	52	218	93	368	4.07	Good
Stability	2	4	76	220	66	368	3.93	Good
Attention	0	5	75	218	70	368	3.96	Good
Total score							19.88	
Average							3.98	
Standard deviation							0.11	
Range							3.87-4.09	
Categories							Good	

The influence of TQM is an endogenous variable 1, which is an approach in adapt of an organization. Respondents' feedback on influence of TQM (Y) has a mean value of 4.02 in good category, it is known that organizational leadership dimension 4.02 is in good category, customer satisfaction and relationship dimension 4.06 is in good category, and human resource management dimension 3.96 is in good category, the strategic planning and development dimension 4.04 is in the good category, and the supplier management dimension 4.00 is in the good category.

Table 6. Total quality management (Y).

Dimension	Response						Average	Categories
	1	2	3	4	5	Total		
Organization leadership	1	4	66	212	85	368	4.02	Good
Costumer satisfaction and relationship	0	3	61	214	90	368	4.06	Good
Human resource management	1	6	74	213	74	368	3.96	Good
Strategic planning and development	0	3	62	220	83	368	4.04	Good
Supplier management	0	6	64	223	75	368	4	Good
Total score							20.08	
Average							4.02	
Standard deviation							0.03	
Range							3.98-4.05	
Categories							Good	

Correlation coefficient analysis was used to determine the degree of intimacy of the relationship between the independent variables. Using the application assistance of the SPSS program, the output of the correlation coefficients of leadership, technical competence and organizational culture are as follows, as shown in the following *Table 7*.

Table 7. Correlation coefficients between correlation independent variables.

		Leadership (X_1)	Technical Competence (X_2)	Organizational Culture (X_3)
Leadership (X_1)	Pearson correlation	1	0.668**	0.701**
	Sig. (2-tailed)		0.000	0.000
	N	368	368	368
Technical competence (X_2)	Pearson correlation	0.668**	1	0.655**
	Sig. (2-tailed)	0.000		0.000
	N	368	368	368
Organizational culture (X_3)	Pearson correlation	0.701**	0.655**	1
	Sig. (2-tailed)	0.000	0.000	
	N	368	368	368

Questionnaire data processing with SEM, the communicator dimension has the largest loading factor compared to other dimensions of 0.90. It can be interpreted that the communicator dimension makes the greatest contribution to enhancing leadership. The expertise dimension has the largest loading factor compared to the other dimensions, which is 0.86. This means that the expertise dimension makes the greatest contribution to improving the technical competence of operating employees. The yield orientation and stability dimension had the largest loading factor compared to the other dimensions, namely 0.85. It can be interpreted that the dimensions of orientation and decision stability make the greatest contribution in improving organizational culture. The dimension of organizational leadership and strategic planning and development has the largest loading factor compared to other dimensions of 0.93. This means that the dimensions of organizational leadership and strategic planning and development make the greatest contribution in improving the influence of TQM. And finally, the productivity dimension has the largest loading factor compared to the other dimensions, which is 0.91. It can be interpreted that the productivity dimension makes the biggest contribution in improving the influence of TQM in the floor tiles industry as shown in the following *Table 8*.

Table 8. Correlation coefficients between correlation independent variables.

Factor	Dimension	Loading Factor	t-Value	Error	Information
Leadership (X_1)	Director ($X_{1.1}$)	0.84	19.41	0.29	Valid
	Communicator ($X_{1.2}$)	0.90	21.62	0.19	Valid
	Decision making ($X_{1.3}$)	0.80	10.11	0.35	Valid
	Motivator ($X_{1.4}$)	0.89	21.20	0.21	Valid
Technical competence (X_2)	Knowledge ($X_{2.1}$)	0.79	17.50	0.38	Valid
	Skill ($X_{2.2}$)	0.86	19.90	0.27	valid
	Attitude ($X_{2.3}$)	0.83	18.92	0.31	Valid
Organizational culture (X_3)	Innovation ($X_{3.1}$)	0.8	17.92	0.37	Valid
	Result orientation ($X_{3.2}$)	0.85	19.71	0.29	Valid
	Aggressive ($X_{3.3}$)	0.83	19.18	0.31	Valid
	Stability ($X_{3.4}$)	0.85	19.98	0.28	Valid
	Attention to details ($X_{3.5}$)	0.8	18.18	0.36	Valid
TQM (Y)	Organization leadership (Y_1)	0.93	0.00	0.20	Valid
	Costumer satisfaction & relationship (Y_2)	0.92	24.77	0.15	Valid
	Human resource management (Y_3)	0.91	23.99	0.18	Valid
	Strategic planning and development (Y_4)	0.93	25.62	0.13	Valid
	Supplier management (Y_5)	0.9	23.72	0.19	Valid

In this analysis, the value of the model fit indicator (fit index) as a LISREL output will be discussed. In this analysis, to see whether the model obtained has met the model accuracy measure Goodness of Fit (GoF) measures so that it can be said that the model obtained from the comparison between the data and the model is good, it can be seen based on the following criteria.

Table 9. Results of analysis of variable measurement model.

No	Indicator	Standard	Value	Model Fit Criteria
1	Chi-square	< 2df	212.89 < 2 -182	Model Fit
2	Probability (p-value)	≥ 0.05	0.0582	Model Fit
3	Root Mean Square Error of Approximation (RMSEA)	≤ 0.08	0.036	Model Fit
4	Normed Fit Index (NFI)	≥ 0.90	0.98	Model Fit
5	Comparative Fit Index (CFI)	≥ 0.90	1.00	Model Fit
6	Incremental Fit Index (IFI)	≥ 0.90	1.00	Model Fit
7	Goodness of Fit Index (GFI)	≥ 0.90	0.96	Model Fit

Based on the *Table 9*, it appears that in general goodness of fit requirement has been met because the values obtained are within the required interval, so that it is said that the model obtained is fit. The results of the model accuracy calculation (suitability benefit measure) show the leadership, technical competence, and organizational culture model on influence of TQM and its effect at floor tiles industry is a good model to explain the relationship of variables studied.

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Table 10. Direct and indirect effects of leadership (X_1), technical competence (X_2) and organizational culture (X_3) on TQM (Y).

	Path Coefficient	Direct Effect	Indirect Influence Through: Leadership (X_1)	Technical Competence (X_2)	Organizational Culture (X_3)	Total Effect
Leadership (X_1)	0.17	2.89%		3.31%	9.69%	15.89%
Technical competence (X_2)	0.21	4.41%	3.31%		11.13%	18.86%
Organizational culture (X_3)	0.62	38.44%	9.69%	11.13%		59.26%
		45.74%	13.00%	14.45%	20.82%	94.00%
Total effects of X_1, X_2 and X_3 on Y						

Based on the table above, it can be seen that leadership, technical competence and organizational culture have a direct and indirect impact on the influence of TQM in the floor tiles industry. From the recapitulation of the influence between variables, it is found that the direct influence of leadership on TQM implementation is 2.89%, indirect effect through technical competence is 3.31% and indirect influence through organizational culture is 9.69%, so the total influence of leadership on TQM is 15.89%. The direct influence of technical competence on the of TQM is 4.41%, indirect influence through leadership is 3.31%, indirect influence through organizational culture is 11.13%, total of influence of technical competence on TQM is 18.86%. The direct influence of organizational culture on TQM is 38.44%, indirect influence through leadership is 9.69%, indirect influence through technical competence is 11.13%, and total of influence of organizational culture on TQM is 59.26%. Thus the proposed conceptual hypothesis has been tested and is acceptable. The computational results obtained show that the total variables of quality management implementation are influenced by leadership variables, technical competence variables and organizational culture variables either partially or simultaneously.

5 | Conclusion

Based on the calculation, the value of $F_{calculate}$ is 1900.889 where reject criterion is H_0 if $F_{calculate}$ is greater than F_{Table} or $F_0 > F_{Table}$, with degrees of freedom $v_1 = 3$ and $v_2 = 368–3–1$ and 95% confidence level, then from our F distribution table get the value of F_{Table} for F0.05,3,368 = 2.6484. Since 1900.889 is greater than 2.6484, then H_0 is subtracted, meaning it can be concluded that there is a positive influence between leadership variables, technical competence variables and organizational culture variables on the influence variables of in TQM. Based on the components that influence of TQM in floor tiles industry, the variables of leadership, technical competence and organizational culture are closely related to certain factors. A leadership pattern that focuses on goals or targets determined and taken based on scientific thinking by taking into account the various parties involved will improve the quality of influence of TQM in the company. Education and training as well as employee involvement by providing controlled independence as well as improving the technical quality of employees is useful to maximize the influence of TQM. Similar to leadership and technical competencies, an organizational culture that focuses on customers, is committed, supports each other or works collaboratively and has a desire to maximize all facilities provided for self-empowerment will have a positive impact on the influence of TQM. By supporting the optimization of the variables of leadership, technical competence and organizational culture, the influence of TQM in floor tiles industry can be optimized as well. On the other hand, lack of attention to the aspects of leadership, technical competence and good organizational culture will hamper the influence process of TQM.

Conflicts of Interest

This statement is to certify that all authors have seen and approved the manuscript being submitted. We warrant that the article is the authors' original work. We warrant that the article has not received prior publication and is not under consideration for publication elsewhere. On behalf of all co-authors, the corresponding author shall bear full responsibility for the submission. This research has not been submitted for publication nor has it been published in whole or in part elsewhere. We attest to the fact that all Authors listed on the title page have contributed significantly to the work, have read the manuscript, attest to the validity and legitimacy of the data and its interpretation, and agree to its submission to the journal of applied research on industrial engineering.

All authors agree that author list is correct in its content and order and that no modification to the author list can be made without the formal approval of the editor-in-chief, and all authors accept that the editor-in-chief's decisions over acceptance or rejection or in the event of any breach of the principles of ethical publishing in the journal of applied research on industrial engineering being discovered of retraction are final.

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Effect of Fibre Parameters on the Physical and Mechanical Properties of Epoxy-Based Reinforced Deleb Palm Fibre Composite; Using Taguchi Grey Relational Optimization

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Abstract

This study considered the consequence of the length and weight composition percentage of Deleb palm fruit fiber on the physio-mechanical characteristics of an epoxy-based composite through the Taguchi grey relational optimization technique. Considering fiber reinforcement of 30 wt% to 40 wt% and fiber Reinforcement Length (RL) of 1-5 mm, the physical and mechanical properties were determined based on standards. The findings demonstrated that the Deleb palm fruit fiber's characteristics tend to differ from those of other types of fiber reinforcement in that they significantly impact the physio-mechanical characteristics of the resulting epoxy-based reinforced Deleb palm fruit fiber composite. The Analysis of Variance (ANOVA) result showed that, at a confidence interval of 5%, the effects of the fiber characteristics on the physio-mechanical properties of the composites were particularly notable for tensile strength and a decrease in water absorption.

Keywords: Composites, Deleb palm fibre, Physio-mechanical properties, Taguchi, Optimization.

1 | Introduction

There is a constant rise in concern for environmental sustainability, with rising worldwide environmental consciousness, as a result of the unavoidable expanding global waste problem, whose existing management strategy poses a threat to the ecosystem through pollution of the water, air, food, and life according to Lohman [1] and Mortaz Hejri et al. [2]. Numerous research has been done on this subject because of the rise in the manufacturing and usage of plastic, the price of resin, and growing environmental safety concerns [3]. Although as a result of the global quest for an eco-friendly environment devoid of pollution and harmful gases, man has acceded to better waste management techniques, such as turning waste into money by combining agricultural and polymer wastes into valuable products. Due to this, it has gained prominence in global research [4]. The desire for new materials has increased due to man's ongoing pursuit of technological advancement. Composite

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materials, for example, surpass monolithic materials, which previously dominated the market for technological materials. Agro waste can be used for a variety of things, such as animal feed, fuel, and filler materials. This is especially true if processed into particles or fibers that are valuable raw materials for complex composites [5]. There is great potential for natural fiber-reinforced composite [6].

Even though natural fibers have a better stiffness and strength-to-weight ratio than traditional reinforcing materials, they have received much attention in the last 20 years as a replacement for synthetic fiber reinforcements like glass and carbon [7]. Recycled natural fiber materials have been demonstrated to offer superior composite properties in some cases, making them useful in various industries [7]–[9]. Depending on the sources and methods of extraction used, their quality can vary [10]. Natural fibers are advantageous because they are readily available, inexpensive, low-density, and environmentally benign. However, employing them has a disadvantage. Natural fibers tend to soak up a lot of moisture, influencing how well they adhere to the matrix. Fortunately, chemical processing can make the fiber's surface characteristics better.

Tropical Africa is the natural habitat of the sturdy, single-stemmed Deleb palm tree. The unbranched stem can reach a height of 20 to 30 meters and a thickness of 40 to 50 centimeters. Usually, the base of the palm is wider than the stem and has a thickness of 85 cm. The middle of the tree enlarges as it ages, eventually growing to a diameter of 80 cm. It has enormous, 3 to 4-meter-long, and 3 to 4-meter-wide, fan-shaped leaves with spines on the tips. A helpful tree, the Deleb palm, is used for food, medicine, and other things [11]. This study reinforces a polymer-based composite with the fiber husk of its fruit by utilizing the multiple economic, technological, and environmental advantages of natural fibers.

One of the most adaptable types of thermoset, epoxy resins, finds use in various industries, including composites, wind energy, building, electrical, and paint and coating [12]. The monomer of the resins contains two or more epoxy groups that resemble rings. A class of reactive polymers and prepolymers with an epoxide in them includes epoxy resin [12]–[14]. Epoxies are amorphous resins that can be adjusted to attain glass transition temperatures in the range of 60^{o}C up to 250^{o}C, depending on the choice of ingredients, using various prepolymers. The qualities may be hard and powerful or tough and resilient. The compressive strength of thermosets is twice as strong as the tensile strength, which can be among the highest values possible and occasionally surpass 80 MPa. With elongation-to-break values typically in the range of 5%-10%, high strength can be paired with a practical level of ductility [12]. In order to help the composite solidify, epoxy resins are frequently employed in conjunction with a hardener, a viscous liquid. Epoxy resin transforms into a shelf-stable liquid when the curing agent is introduced.

Natural fiber reinforced polymer composites have emerged as a potentially eco-friendly, economical, and practical material [15]–[18]. According to Onyekwere et al. [18], [19], by utilizing characteristics like low density, great specific strengths ands moduli, and affordable costs of natural fibers over synthetic fibers, Natural Fiber Composites (NFC) can find a wide range of uses in the creation of valuable materials. Even so, a significant concern that needs to be addressed is the challenge of its poor matrix compatibility, high natural fiber water absorption rate, and low recycling rate [20]. For instance, a considerable amount of milled fiber (50–70 wt percent) was used to create epoxy-peach palm tree fiber composites [21]. It was shown that this reinforcement significantly increased the composite's modulus in the rubbery area. Synthetic fiber-reinforced epoxy composites (with synthetic fibers such as carbon, glass, aramid, and kevlar) are typically employed in automotive and construction applications because of their lightweight, strong strength, and good modulus; however, natural fibers are seen to be a better choice because it is uncertain whether the synthetic fibers from composites can be recycled after usage. Yet a number of elements—including filler characteristics, filler shape, matrix characteristics, filler orientation in the matrix, filler-matrix interactions, and filler volume fraction—are linked to the performance of the composite [20].

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No standard parameter setting for processing NFC applies to all-natural fibers since different types of natural fibers have distinct processing properties. As a result, Deleb palm fruit fiber processing parameters must be optimized. This is because choosing the best natural fiber matrix combinations and processing techniques will require determining the best parameter values for high-performance composites [18], [22]. This study aims to ascertain the impact of fiber characteristics on the physio-mechanical characteristics of an epoxy-based composite reinforced with Deleb palm fruit fiber. Additionally, optimizing the fiber characteristics of Deleb palm fruit fiber using Taguchi grey relational optimization is studied for the first time in producing epoxy resin composite materials to enhance the following physical and mechanical qualities.

2 | Materials and Methods

2.1 | Materials

The under-listed materials were used for this research work.

I. Deleb palm fruit consists of natural fiber from the Deleb palm fruit (Borassus Aethiopum) acquired at Zaria, Nigeria.
II. Epoxy resin: the Nigerian institute of leather and science technology Samaru in Zaria, Nigeria is where the epoxy resin was purchased. The details included a light yellow appearance, the equivalent epoxy weight of 187 g/eq, and room temperature viscosity of 12600 cP.
III. Hardener: in this study, Hexamethylenediamine was used, a popular hardener.

2.2 | Experimental Design

Based on recommendations from previous studies [22], [23] this study used Taguchi's Orthogonal Array (OA) to design the experiments. Two processing factors, each with three levels of design, were adopted for this study. *Table 1* shows the factors and classes used to develop the reinforcement and matrix composites. In contrast, *Table 2* shows the orthogonal array of L9 generated in the Minitab statistical software.

Table 1. Factors used in the development of the composites.

Factors	Levels		
	1	2	3
Reinforcement Weight (RW) percentage of fiber (%wt)	30	35	40
Reinforcement Length (RL) (mm)	1	3	5

Table 2. Experimental layout.

Experimental Number	RW (%wt)	RL (mm)
RWL1	30	1
RWL2	30	3
RWL3	30	5
RWL4	35	1
RWL5	35	3
RWL6	35	5
RWL7	40	1
RWL8	40	3
RWL9	40	5

2.3 | Composite Formulation

The fiber was taken from the mesocarp, the fruit's outermost layer of the Deleb palm. Retting was employed in the extraction procedure to separate the fibers from the woody core. The fibers were extracted from the Deleb palm shells by pulling them out, and they were then washed to remove the embedded dirt between the fibers and allowed to air dry for 48 hours. In order to lessen the effects of variance, the fibers were carefully mixed throughout the extraction process. Fiber treatment, one way of increasing the compatibility of natural fiber with the hydrophobic polymer matrix [18], was employed. Before the alkali treatment, the coir fibers were chopped into the appropriate diameters. *Eq. (1)* was used to prepare the solution [19], [24], [25]:

$$\mathrm{NaOH} = \frac{\text{percentage needed}}{100} \times \text{Total volume of distilled water.} \tag{1}$$

After submerging in an alkaline solution with a 6% concentration for 3–4 hours, the fibers were repeatedly rinsed with water to remove any remaining sodium hydroxide. According to the recommendations of Yan et al. [26], the fibers were then oven-dried for 8 hours at 70°C.

After the fibers were prepared, they were put within an 80 by 30 by 5 mm3 mold covered in a release spray. According to the hand lay-up method, the resin and hardener mixture was gradually poured over the fiber as further layers of fiber were added to one another to reach the desired thickness. Following production, the samples underwent a 5-minute curing process at 150°C and 2.5 MPa of mold pressure. *Fig. 1* displays the created samples for this study.

Fig. 1. Developed samples.

2.4 | Characterization

2.4.1 | Tensile properties

According to based on American Society for Testing Method (ASTM) D-638, the tensile test was performed using a Hounsfield Monsanto Tensometer (model 9875). Tensile parameters for each sample, including tensile strength, strain, and modulus, were determined after a tensile force was applied to dumbbell-shaped samples using *Eqs. (2)* to *(4)*.

$$\text{Tensile Strength} = \frac{\mathrm{F}}{\mathrm{bd}}. \tag{2}$$

$$\text{Strain} = \frac{\Delta \mathrm{L}}{\mathrm{L}}. \tag{3}$$

$$\text{Modulus} = \frac{\text{Stress}}{\text{Strain}}. \tag{4}$$

b is the sample thickness, d is the sample breadth, L is the change in length, L is the gauge length, and F is the maximum tensile force.

2.4.2 | Flexural strength

The load in the middle of the two endpoints determines how the material responds, referred to as the material's flexural properties. The sample was built in a rectangular shape with a 5.0 mm thickness using the motorized automatic recording Tensometer for the flexural strength test ASTM D7028-07-2015. The autographic recording drum was then taken out of the machine and coated with graph paper designed especially for the drum. The sample was fixed to the Tensometer using the flexural fixture and pins. The autographic recording drum was turned to the proper starting position, and the perspex indicator and the pricker of the graph sheet were both set to zero. As the load was applied to the sample, the amount of elongation or deflection was continually transmitted via a rotating spindle until failure. Using a sliding arm to follow the column and a button to mark the graph sheet at certain intervals, the load was then drawn on the deflection diagram. After the test, the machine's autographic recording drum and the graph sheet paper were removed. On graph paper, the force and deflections were measured for analysis. The flexural strength was evaluated according to *Eq. (5)* [27].

$$\sigma = \frac{3FL}{2bd^2}. \tag{5}$$

L is the length of the support span, F is the force at the fracture point, and b and d are the breadth and thickness, respectively.

2.4.3 | Impact energy

The materials were tested for impact energy using a Ceast Lot-Resil Impactor with Ceast NotchVIS Unit at the Nigerian Institute of Leather and Science Technology Samaru, Zaria, Nigeria. The sample was created per the DENT-specific ISO 8256 (2004) standard. These measurements: 64 mm x 12.7 mm x 5 mm, were used to conduct the impact study. Up to an early crack length of 2 mm, which implies 1 mm on each side, the sample's thin sidewalls were nicked with metal blades using a pneumatic notching tool. The test was conducted following ISO 291 (2008) standards, with an ambient temperature of 23°C and relative air humidity of 50%, as detailed in the study of Salakhov et al. [28].

The sample was secured in a clamp parallel to an immovable clamp, and the pendulum hammer's crosshead impacted the sample at the lowest point of the circular motion. With the notches in the middle, the gauge length l0 mm was initially 30 mm. The hammer speed corresponding to this was 3.7 m/s. Following that, the hammer speed was set to 1.5 m/s, or a 60° falling angle. According to the pendulum device's service manual, the recorded load values were analyzed using extension diagrams.

2.4.4 | Water absorption characteristics

Since the natural fibers used in the composite's development are bio-based and have a propensity to absorb water, monitoring their water absorption characteristics over time is essential for the best effectiveness. In fiber-reinforced polymer composites, water absorption can lead to matrix cracking, dimensional instability, and poor mechanical properties [29]. In accordance with ASTM standard D 570-98, the water absorption test involved measuring the samples' differences before and after exposure to water at regular intervals. *Eq. (6)* provides the formula needed to determine the values:

$$\text{Moisture Content} = \frac{M_2 - M_1}{M_1} * 100\,. \tag{6}$$

M_1 is the composite's initial weight, and M_2 is the composite's final weight after immersion.

2.5 | Numerical Analysis

The quantitative analysis employed in this study was grey relational analysis. The grey relational analysis begins with the development of grey relations. At this point, the gathered responses were standardized

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between zero and one. The desired and actual responses were connected using the Grey Relational Coefficients (GRCs) derived from the normalized data [29]. Next, the average of the GRCs for each performance criterion was used to determine the grey relational grade. The grey relational grade was utilized to evaluate the overall effectiveness of the numerous performance metrics. As a result, the optimization of a complex set of various performance indicators was changed into the optimization of a single grey relational grade [30]–[32]. The ideal level for a process parameter was the one with the highest grey relational grade. Additionally, an Analysis of Variance (ANOVA) was utilized to establish the statistical significance of the process components.

Eq. (7) was used to calculate the greater-is-better criterion for the linear data preprocessing method used in this study for the physical and mechanical features of the generated composite. *Eq. (9)* calculated the average of each response variable of the produced composites, and *Eq. (8)* was used to calculate the GRC. x_i (k) is the preprocessed data, min y_i (k) is the smallest y_i (k) estimation for the response, k^{th} and max y_i (k)

$$x_i\ k) = \frac{y_i\ k) - \min y_i(k)}{\max y_i\ k) - \min y_i(k)}, \tag{7}$$

is the largest y_i (k) estimation for the response, k^{th}.

$$\xi_i\ k) = \frac{\Delta_{min} + \zeta\Delta_{max}}{\Delta_{oi}\ k) + \zeta\Delta_{max}}. \tag{8}$$

ζ is the differentiating coefficient (0~1), customarily assigned an equal weight of 0.5 to each parameter; and $\Delta_{oi}\ k) = \|x_o\ (k) - x_i\ (k)\|$, in which $x_o^*(k)$ and $x_i^*(k)$ denote the reference and similarity arrangements, respectively. Every reaction variable's base and mainly deviations are called Δ_{min} and Δ_{max}.

$$\gamma_i = \frac{1}{n}\sum_{i=1}^{n} \xi_i\ k)\ . \tag{9}$$

γ_i is the value of GRG determined for the ith experiment, and n is the total number of performance characteristics.

3 | Results and Discussion

3.1 | Tensile Properties

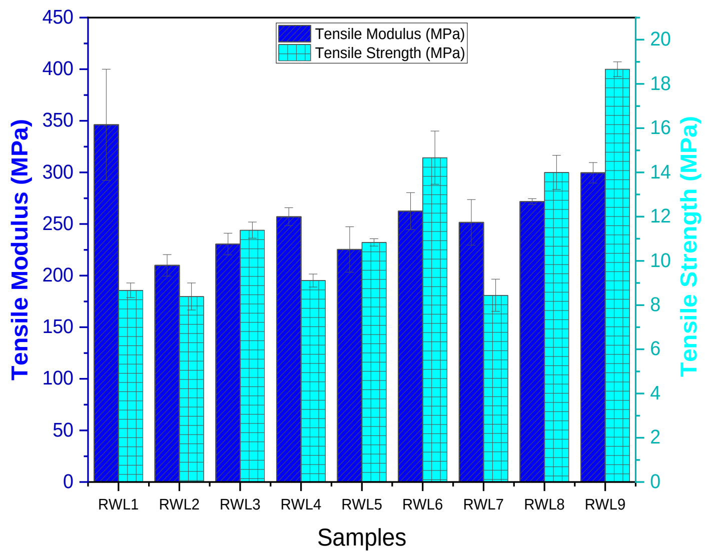

Fig. 2. Tensile modulus and strength of the developed composites.

Fig. 2 shows the tensile modulus of the developed samples. In comparing the tensile moduli of samples, it was observed that with an increase in the RL from 1 mm to 5 mm, there seems to be a reduction in the tensile modulus. This is a result of an increase in the strain of the samples as a result of the presence of reinforcement. One would have expected that the tensile modulus of the composite samples would increase as the fiber length increases, as obtained in other studies like that of Mohammed et al. [29]. However, this result trend can be associated with the elastic property of the Deleb palm fruit fiber, as given by Ngargueudedjim et al. [33].

Additionally, *Fig. 2* displays the generated samples' strength and tensile modulus. In comparing the tensile strengths of the samples, it was found that the tensile strength of the composite increased as the reinforcement's fiber length went from 1 mm to 5 mm. Furthermore, with an increase in the weight percentage of the reinforcement from 30 wt% to 40 wt% in the same sample length, there was an increase in the tensile strength except for sample RWL7, which had a drop in tensile strength in comparison with sample RWL4. This drop can be due to the poor homogeneity of the reinforcement with the epoxy matrix. However, the presence of fiber reinforcement in the polymeric matrix is more noticeable at sample RWL3 (5mm, 40 wt%) with an increase in length and weight percentage, leading to improved tensile strength. The obtained result is in line with the findings of Oladele et al. [34], who reinforced epoxy composites with palm kernel shell fiber and particulate cassava peel.

3.2 | Flexural Strength

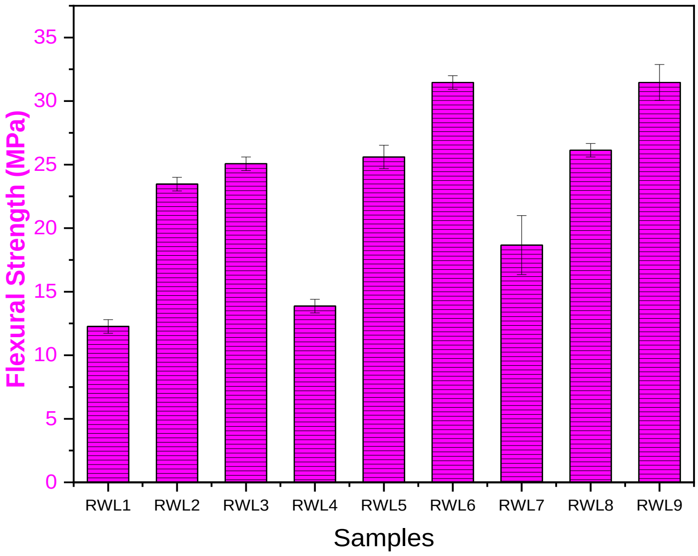

Fig. 3. Flexural strength of the developed composites.

A three-point bending flexural test tested the fiber composite samples, and the result obtained is shown in *Fig. 3*. When the flexural strengths of samples were compared, it was found that the flexural strength of the composite increased with an increase in the reinforcement's fiber length from 1 mm to 5 mm. Furthermore, with an increase in the weight percentage of the reinforcement from 30 wt% to 40 wt% with the same sample length, there was an increase in flexural strength. As to some studies like that of Oladele et al. [32], flexural strength usually increases to a particular level and then decreases. That was ascribed to poor flexural properties of the fibers; however, that was not observed in this case. A similar result was obtained by Davindrabrabu et al. [35], who reinforced polymer material with pineapple leaf fibers.

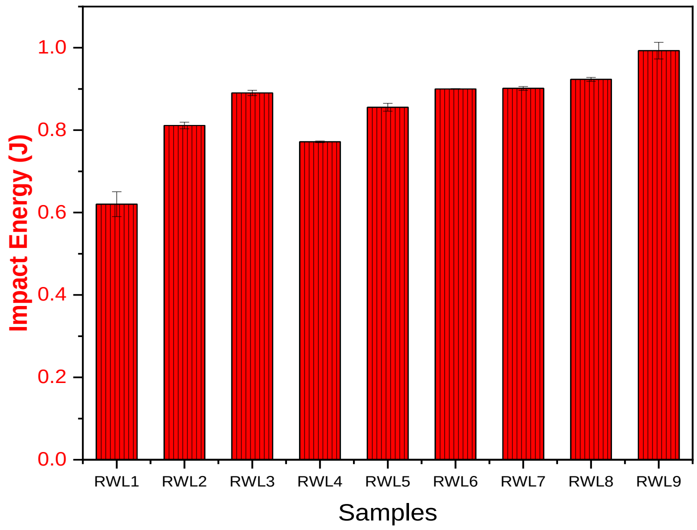

Fig. 4. Impact energy of the developed composites.

3.3 | Impact Energy

The fiber composite samples were tested using the Charpy impact test machine, and the result obtained is presented in *Fig. 4*. When the impact energy of samples was compared, it was found that the impact energy of the composite increased as the reinforcement's fiber length increased from 1mm to 5mm. Furthermore, with an increase in the weight percentage of the reinforcement from 30 wt% to 40 wt% with the same sample length, there was an increase in the impact energy. The high microfibrillar feature of Deleb Palm fruit fiber as a reinforcing material can be credited for this. Studies have shown that reinforcing fibers with higher microfibrillar angles typically have higher impact strength [8]. The obtained result is in line with the findings of Costa et al. [36], who reinforced epoxy with mallow fibers and evaluated the Izod impact and bending properties.

3.4 | Water Absorption Characteristics

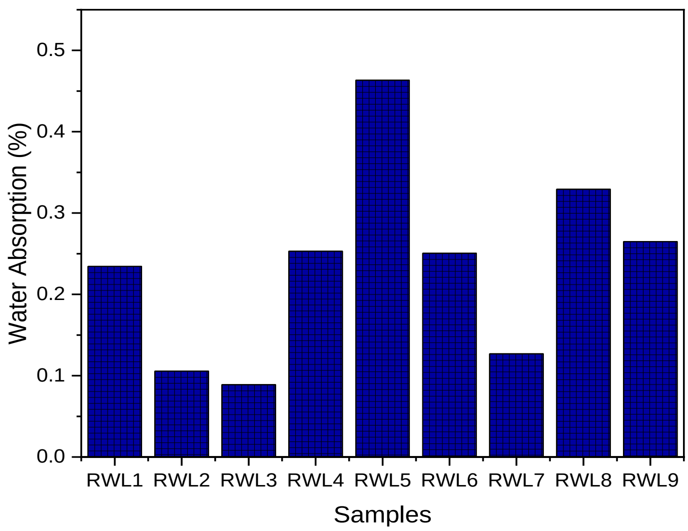

Fig. 5. Water absorption of the developed composites.

In order to ascertain how much water the composite will absorb during a specified period (in this study, seven days), under specific circumstances, the water absorption test was performed. The results are shown in *Fig. 5*. When comparing the water absorption of the samples, it was found that there was a rise in water absorption for each fiber length with a weight percentage increase of the reinforcement from 30% to 40%. At room temperature, it was discovered that the water absorption of the composites followed fiction diffusion. The hydrophilic properties of natural fibers, which are cellulose fibers, can be used to explain this occurrence. However, it was found that once the fiber length was increased over 3 mm, the water absorption started to decrease.

Contrary to the majority of studies on fiber-epoxy composites, which hold that longer fibers can absorb water simultaneously with the fiber and at the fiber/matrix interface without interruption, for shorter fibers, the ability to absorb water is typically hampered by the matrix because the matrix absorbed less water than the fiber did [37]. Therefore, the property of the Deleb palm fruit fiber can be attributed to the decrease in the water absorption property of the composite with an increase in the fiber length [38], making Deleb palm fruit fiber a suitable reinforcing material with desired low water absorption properties that have previously limited epoxy-fiber reinforcement. This highlights some of the several applications for palm fibers.

According to Jeyapragash et al. [39], the mechanical properties of Epoxy-Deleb palm fiber composites developed in this work were compared to those of other natural fiber-epoxy composites, such as tensile strength, flexural strength, and impact energy (as presented in *Fig. 6*). Because of characteristics like cellulose enrichment and increased adherence between natural fiber and epoxy matrix, natural fiber reinforcement alters mechanical properties. The higher impact energy in this study is due to the fiber's high microfibrillar characteristic from Deleb Palm Fruit [11]. High impact strength is typically achieved using reinforcing fibers with high microfibrillar angles [8].

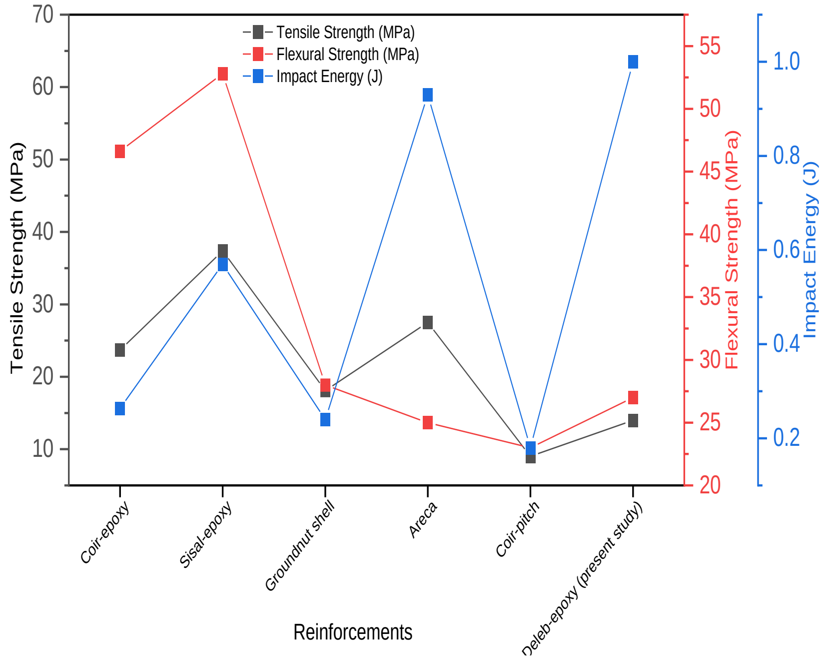

Fig. 6. Comparison of some of the mechanical characteristics of epoxy composites reinforced with natural fiber.

3.5 | Grey Relational Analysis

The responses' data processing (normalization), which was done using the grey relational analysis and the maxim "the more, the better," is shown in *Table 3*. Additionally, *Table 4* provides the gray relational coefficient of each developed sample based on their responses, and *Table 5* provides the grey relational grade appropriate to identify the ideal fiber parameters that produced the best physical and mechanical properties of epoxy-based reinforced Deleb palm fiber composite.

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Table 3. Normalized physio-mechanical properties of the developed composites.

Samples	Tensile Modulus	Tensile Strength	Flexural Strength	Impact Strength	Reduction in Water Absorption
RWL1	1	0.040728801	1.5975E-15	1	1
RWL2	0	0	1.5975E-15	0.614271935	0.898770986
RWL3	0.18753907	0.382243272	1.5975E-15	0.546491137	0.611596593
RWL4	0.40468511	0.10325501	6.94566E-16	0.207437561	0.612123717
RWL5	0.141690795	0.319717063	0.3731143	0.660684278	0.530642173
RWL6	0.446488028	0.698483855	1.5975E-15	0	0.50580553
RWL7	0.361439705	0.008252595	1	0.441400039	0.568713571
RWL8	0.516119544	0.640321573	0	0.452072814	0
RWL9	0.711088869	1	0.660876372	0.882419386	0.208956187

Table 4. GRC of the physio-mechanical properties of the developed composites.

Samples	Tensile Modulus	Tensile Strength	Flexural Strength	Impact Strength	Reduction in Water Absorption
RWL1	0.333333333	0.924677952	1	0.333333333	0.333333333
RWL2	1	1	1	0.448723498	0.357456657
RWL3	0.727231399	0.566737107	1	0.477787133	0.449803466
RWL4	0.55267849	0.82883688	1	0.706776156	0.449590268
RWL5	0.77919148	0.609966563	0.572662709	0.430780368	0.485134427
RWL6	0.5282687	0.417193772	1	1	0.49711399
RWL7	0.580423676	0.983762808	0.333333333	0.531123836	0.467852204
RWL8	0.516119544	0.640321573	0	0.452072814	0
RWL9	0.711088869	1	0.660876372	0.882419386	0.208956187

Table 5. GRC of the physio-mechanical properties of the developed composites.

Samples	Grey Relational Grade	Ranking
RWL1	0.584936	6
RWL2	0.761236	1
RWL3	0.644312	5
RWL4	0.707576	2
RWL5	0.575547	8
RWL6	0.688515	4
RWL7	0.579299	7
RWL8	0.691142	3
RWL9	0.448768	9

From the ranking of the grey relation grade, as shown in *Table 5*, it was observed that sample RWL2 denoting 30 wt% compositions of the Deleb palm fruit fiber reinforcement at 3mm fiber length gave the optimum physio-mechanical properties. This result is supported by the main effects plot for the SN ratio obtained from the Minitab Statistical software for Taguchi design of experiment analysis (*Fig. 7*).

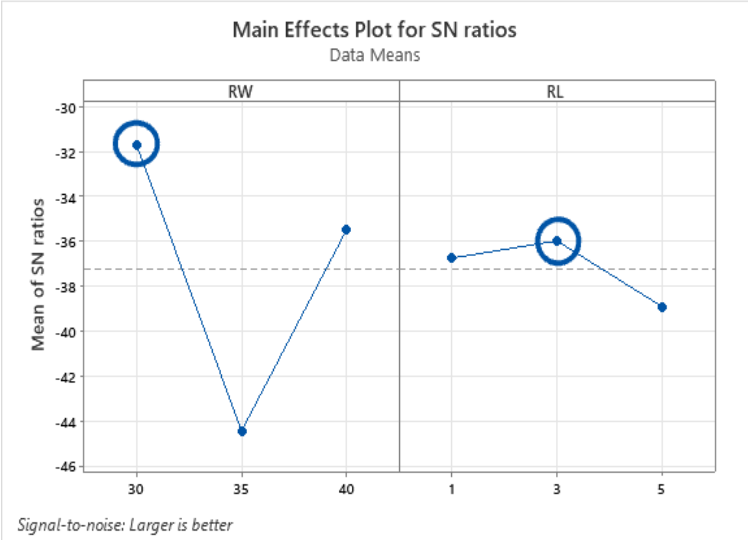

Fig. 7. Main effects plot for SN ratio denoting the optimal composite.

3.6 | Analysis of Variance

To ascertain the relevance of the performance characteristics, a statistical study known as ANOVA was also conducted. The F-test was performed to identify which factor combination substantially impacts the performance characteristic of each physio-mechanical property. This study provided information on the percentage contribution by each factor combination using the total sum of the squared deviations SST.

Table 6. ANOVA of the fiber parameters' impact on the physio-mechanical characteristics of composites.

Source	DF	Sum of Squares	Mean Square	F-value	P-value	Remark
Response: Tensile Modulus						
RW	1	218.5	218.5	0.10	0.761	Insignificant
RL	1	642.8	642.8	0.30	0.604	Insignificant
Residual error	6	12909.6	2151.6			
Total	8	13770.9				
Response: Tensile Strength						
RW	1	26.74	26.741	9.43	0.022	Significant
RL	1	57.04	57.042	20.12	0.004	Significant
Error	6	17.01	2.836			
Total	8	100.80				
Response: Flexural Strength						
RW	1	1.1874	1.1874	4.04	0.091	Insignificant
RL	1	0.1391	0.1391	0.47	0.517	Insignificant
Error	6	1.7654	0.2942			
Total	8	3.0919				
Response: Impact Strength						
RW	1	0.000039	0.000039	0.35	0.577	Insignificant
RL	1	0.000014	0.000014	0.12	0.740	Insignificant
Error	6	0.000678	0.000113			
Total	8	0.000731				
Response: Reduction in Water Absorption						
RW	1	0.07009	0.070095	22.92	0.003	Significant
RL	1	0.01706	0.017056	5.58	0.056	Insignificant
Error	6	0.01835	0.003059			
Total	8	0.10550				

The tensile strength and water absorption were the two physio-mechanical features of the composites most significantly impacted by the fiber parameters, according to the ANOVA result obtained and presented in *Table 6* with a 95% confidence interval.

4 | Conclusion

In this study, Taguchi grey relational optimization was used to examine the impact of fiber parameters on the physical and mechanical characteristics of an epoxy-based reinforced Deleb palm fiber composite. The results found that the Deleb palm fruit fiber's features tend to differ from those of other types of fiber reinforcement in terms of their impact on the physio-mechanical properties of the epoxy-based reinforced Deleb palm fruit fiber composite.

Additionally, the results demonstrated that sample RWL2, which contained a 30-weight percent composition of Deleb palm fiber reinforcement with a 3 mm fiber length, was the sample with the best physio-mechanical properties. These samples were identified by the ranking of the grey relation grade and the main effect plot of the SN ratio. The ANOVA result showed that, at a confidence interval of 5%, the composites' tensile strength and reduction in water absorption were the most significantly affected physio-mechanical properties by the fiber parameters.

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Conflicts of Interest

The authors report there are no competing interests to declare.

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A Two-Stage Stochastic Programming Approach for Care Providers' Shift Scheduling Problems

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Abstract

Due to the importance of the health field, the problem of determining the shift scheduling of care providers has been addressed in many studies, and various methods have been proposed to solve it. Considering different skills and contracts for care providers is one of the essential issues in this field. Given the uncertainty in patients' demands, it is a crucial issue as to how to assign care providers to different shifts. One area facing this uncertainty is the provision of services to cancer patients. This study develops a stochastic programming model to account for patient demand uncertainty by considering different skills and contracts for care providers. In the first step, care providers are assigned to work shifts, then, in the second step, the required overtime hours are determined. The sample average approximation method is presented to determine an optimal schedule by minimizing care providers' regular and overtime costs with different contracts and skills. Then, the appropriate sample size is 100, determined based on the Monte Carlo and Latin Hypercube methods. In the following, the lower and upper bounds of the optimal solution are calculated. As the numerical results of the study show, the convergence of the lower and upper bounds of the optimal solution is obtained from the Latin Hypercube method. The best solution is equal to 189247.3 dollars and is achieved with a difference of 0.143% between the upper bound and lower bounds of the optimal solution. The Monte Carlo simulation method is used to validate the care provider program in the next stage. As shown, in the worst case, the value of the objective function is equal to 197480 dollars.

Keywords: Healthcare, Shift scheduling, Uncertainty, Stochastic programming, Sample average approximation.

1 | Introduction

The due to the increase of chronic diseases such as Covid-19, the costs of healthcare systems are increasing dramatically [1]. Many countries in the world have experienced Covid-19, and as a result, the need for care providers with different skills has increased [2]. Hospitals in the private sector compete to provide better services at lower costs [3]. The statistics issued by the world health organization indicate that personnel planning will be an essential priority in the field of health in the next decade. Due to the high cost of staffing, which accounts for about 40% of the total costs, medical centers must reduce their expenditures. In this regard, proper human resource planning will help significantly. A practical issue in healthcare is the planning of service shifts. This subset of employee scheduling varies greatly depending on the type of job, employee skill, and work regulations [4]. Nurse Rostering Problem (NRP) or Nurse Scheduling Problem (NSP) aims to create a schedule and assign available service providers to hospital shifts under various constraints over a period [5]. The

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significance of this issue for improving service quality, staff satisfaction, health conditions, and reducing hospital costs has encouraged researchers to study it. Solving this problem results in a schedule that specifies how many people are required for different skills and when they can offer their service on a given planning horizon. The program must comply with labor laws, employee preferences, availability, labor demand, workload and demand, employment contracts, and ergonomic and technical constraints.

The high variety of problems in modeling, making assumptions ,and solving methods have increased the attractiveness of those problems. In the home care field for cancer patients, shift planning is of particular importance due to different contracts for service providers, the high cost of specialized services in the field, and the high uncertainties in patients' conditions. Considering the uncertainty in patients' demands removes one of the important obstacles in making the right decision [6]. In the Iranian Health Control Center, care providers' contracts with different skills are set in three modes: full-time, part-time, and hourly. In addition, the service cost is increased by 20% to 40% per hour as a contract change from full-time to part-time or hourly. Therefore, planning for the proper assignment of care providers can significantly save system costs. As is the case, most medical centers use manual planning, and the Iranian Health Control Center is no exception [7].

Considering the patient's demand uncertainty and different contracts of care, providers can provide an efficient and flexible schedule. A two-stage stochastic programming model is presented to account for these issues. The first stage minimizes the cost of assigning care providers to shifts with different contracts. The amount of overtime required for each skill on each shift is determined in the second stage. Because of the very high number of scenarios, the Sample Average Approximation (SAA) method is used to solve the presented model. Since the sample size is one of the critical parameters of the SAA method, for determining the sample size, the two methods of Monte Carlo and Latin Hypercube are compared, and the results are shown. Based on the best sample size, the care providers' shift planning in the Iranian Health Control Center is presented.

2 | Literature Review

Medical staff planning has been a topic of research since the 1950s. According to Ernst et al. [8], making a schedule that can satisfy employees' needs is not easy. The task of medical staff planning is often complicated by staffing requirements as well as government and hospital regulations. Planners should consider the conditions and the number of patients, the expertise, work experience, and preferences of the medical staff, the hospital policies, and the rules and regulations set by the government [8]. Considering the significance of medical staff scheduling in healthcare, more studies on this issue have been published over the last two decades. Klinz et al. [9] proposed two mathematical models to minimize the total number of work shifts and nurses' general unhappiness. Topaloglu and Selim [10] introduced a multi-objective integer program for NSPs to produce an equitable schedule for nurses and satisfy hospital management objectives. Landa-silva and Le [11] presented a multi-objective approach to cope with real-world uncertainties in NSPs. To do so, they proposed an evolutionary algorithm to achieve high-quality non-dominated schedules. Ohki [12] established a cooperative genetic algorithm to re-optimize nurse schedules. Zhang et al. [13] presented a hybrid and swarm-based optimization algorithm. It combined a variable neighborhood search and a genetic algorithm to cope with a highly-constrained NSP in modern hospitals. Maenhout and Vanhoucke [14] studied the nurse allocation issue and used the column generation method to deal with it. Santos et al. [15] introduced cutting as a concept in integer programming to solve related problems innovatively. Ingels and Maenhout [16] considered the effects of defining and including reserve duties in rosters of medium-term shifts for the personnel. They used a three-stage method that imitated the workforce management process to measure the robustness achieved. After the personnel roster was designed, the events that unexpectedly occurred would be simulated, and an optimization model would determine the adjustments required to balance supply and demand. Bagheri et al. [17] introduced a stochastic mathematical model for an NSP in a heart surgery center to minimize the regular and overtime assignment costs. They assumed that patients' demands and length of stay would be uncertain. So, they used a SAA method to solve the model. In another study, Punnakitikashem et al. [18] sought to minimize the overload of nurses through an integer MP model of a stochastic type. They dealt with the model's staffing cost as a hard budget constraint. Moreover, they used Benders' decomposition and Lagrangian relaxation methods to obtain non-dominated solutions. The resulting model was implemented in two medical and surgical wards at the Northeast Texas hospital. Chen et al. [19] studied an integrated problem of allocating a medical staff and scheduling a general staff under uncertain conditions. They solved the problem by employing a double-stage algorithm to determine a medical staff with the smallest possible size and make the best schedule. Ang et al. [20] introduced a decision support system based on a goal programming method for NSPs. They examined workload distribution, shift equity, and staff satisfaction. They also pursued minimizing the Nurse-Patient Ratio (NPR) calculated based on the number of patients allocated to each nurse. Hamid et al. [21] devised a mathematical model with multiple objectives to schedule a nursing staff, which took the decision-making styles of nurses into account. The objectives addressed in that study were the minimization of the total cost of staffing, minimization of the average index of the incompatibility in the decision-making styles of the nurses assigned to the same shift days, and maximization of the overall satisfaction of nurses with their shifts. Moreover, three meta-heuristics were developed to solve the problem, including the multi-objective Keshtel algorithm, non-dominated sorting genetic algorithm II, and multi-objective tabu search. Hassani and Behnamian [22] developed a sustainable approach with a robust scenario-based optimization method. They proposed the Differential Evolution (DE) algorithm to solve the problem and compared the performance against the genetic algorithm. The results show that the DE algorithm has good performance. Kheiri et al. [23] studied the multi-stage nurse rostering formulation. They proposed a sequence-based selection hyper-heuristic using a statistical Markov model and an algorithm for building feasible initial solutions. Empirical results and analysis show that the suggested approach has significant potential for difficult problem instances. A brief classification of the models reviewed in the literature is presented in *Table 1*. According to a comprehensive literature review, the issue of different contracts for care providers has not been addressed. However, various uncertainties must be addressed in real-world shift scheduling to provide a high-quality schedule. In this study, the subject of the uncertainty of patient demand is considered. To fill this research gap, in this study, the uncertainty of patients' demands and the types of service providers' contracts and skills are used as a basis to develop a two-stage stochastic programming model. In the following, the model is solved with the SAA method, and the parameters of the solution method are adjusted. In the end, the obtained planning validity is shown using the simulation method.

The rest of this article is as follows; Section 3 presents the proposed optimization model and describes its structure. The solution approach is introduced in Section 4, and detailed descriptions are provided for the SAA method. Section 5 presents the statistical experiments. Finally, the concluding remarks are made in Section 6.

Table 1. A brief review of the litreture.

Author	Objective	Constraints					Uncertainty			Solution Approach
		Night Shifts	Working Days	Shift Tyoe	Skill Type	Type of Cntracts	Fuzzy	Stochastic	Robust	
Klinz et al. [9]	Minimizing the total number of work shifts and the general unhappiness of all nurses	✓	✓	✓	-	-	-	-	-	Heuristic
Topaloglu and Selim [10]	Minimizing deviations from nurse preferences and hospital regulations	✓	✓	✓	-	-	✓	-	-	Exact
Landa-silva and Le [11]	Satisfaction with nurse preferences and work regulations	✓	✓	✓	✓	✓	-	-	-	Meta-heuristic
Ohki [12]	Minimizing the penalty function to evaluate shift schedules	✓	✓	✓	-	-	-	-	-	Meta-heuristic

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Table 1. Continued.

Author	Objective	Constraints					Uncertainty			Solution Approach
		Night Shifts	Working Days	Shift Tyoe	Skill Type	Type of Cntracts	Fuzzy	Stochastic	Robust	
Zhang et al. [13]	Maximizing the quality of objectives concerning the importance of constraints	✓	✓	✓	-	-	-	-	-	Meta-heuristic
Maenhout and Vanhoucke [14]	Minimizing the penalty associated with different types of nurses	✓	✓	✓	✓	✓	-	-	-	Exact
Santos et al. [15]	Minimizing the penalty of assignment	✓	✓	✓	-	-	-	-	-	Heuristic
Ingels and Maenhout [16]	Minimizing the allocation penalty and changing the nurse schedule	-	✓	✓	-	-	-	-	-	Exact & Simulation
Bagheri et al. [17]	Minimizing the regular and overtime hours of nurses	✓	✓	✓	-	-	-	✓	-	Sample average approximation
Punnakitikashem et al. [18]	Minimizing the excess workload on nurses and the cost of staffing	✓	✓	✓	✓	-	-	✓	-	Benders & Lagrangian
Chen et al. [19]	Minimizing the penalty of violation of the soft constraints of nurses' preferences	✓	✓	✓	-	-	-	-	-	Exact
Ang et al. [20]	Minimizing the average and maximal deviations from the target ratios of nurse to patient	✓	✓	✓	-	-	-	-	-	Exact
Hamid et al. [21]	Minimizing the total cost of staffing and the sum of incompatibility among nurses and maximizing the satisfaction of nurses with their assigned shifts	✓	✓	✓	✓	-	-	-	-	Meta-heuristic
Hassani and Behnamian [22]	Minimizing the total cost of allocating shifts to nurses, reserve nurses required, overtime and underemployed costs, and cost of mismatching the nurse preferences	-	✓	✓	✓	-	-	-	✓	Meta-heuristic
Kheiri et al. [23]	Minimizing violation of eight soft constraints	-	✓	✓	✓	-	-	-	-	Hyper-heuristic
Current study	Minimizing the costs of assigning care providers to shifts and overtime hours of care providers	-	✓	✓	✓	✓	-	✓	-	Sample average approximation

The paper's main contributions to the literature are considering different skills and contracts for care providers in NRP and dealing with the Latin Hypercube Sampling in the SAA method.

3 | Problem Definition

Hospitals and health centers should provide the necessary services to patients in common and critical situations such as Covid-19. In recent years, due to the decrease in available care providers and the increase in diseases, the tendency for cooperation between health center managers and researchers to properly plan appropriate services for patients has increased. Therefore, one of the most critical issues is the proper distribution of care providers between work shifts. On the other hand, it is impossible to determine the demand for each skill in many cases accurately. Therefore, a two-stage stochastic programming model is proposed to achieve high-quality planning. Some of the assumptions of the problem are as follows:

I. All care providers have identical skills.
II. Demand behavior is the random variable based on a specific distribution function.
III. Each care provider is only assigned one shift each day.
IV. Each care provider has a specific contract.

After solving the proposed model, the work plan obtained for a five-day horizon can be as follows (*Table 2*):

Table 2. An example of a care provider's schedule.

	Sataurday	Sunday	Monday	Tuesday	Wdenesday
Nurse 1	Morning	Afternoon	Morning	Afternoon	Afternoon
General practitioner 1	Afternoon	Morning	-	Morning	Afternoon
Specialist physician 1	Afternoon	Afternoon	Afternoon	-	Morning

In the proposed mathematical model, care providers are assigned to specific shifts, and the number of overtime hours required in possible conditions is determined. The required duration of each skill per day and each shift (de_{smd}) is stochastic. Care providers have three professions: nurses, general practitioners, and specialist physicians. Contracts are also available in three types, full-time, part-time, and hourly. The stochastic demand model for the problem of scheduling care providers can be formulated with the notations as follows:

Sets

S	Set of skills (xx: nurse, xy: general practitioner, xz: specialist physician).
M	Set of shifts (1: morning shift, 2: afternoon shift).
D	Set of days.
N	Set of contracts (full-time, part-time, hourly).
ξ	Set of scenarios ($\xi = 1,2, \dots, B$).
I_{xx}	Set of nurses.
I_{xy}	Set of general practitioners.
I_{xz}	Set of specialist physicians.

Parameters

aa_{ij}	1, if nurse i is under contract j.
ab_{ij}	1, if general practitioner i is under contract j.
ac_{ij}	1, if specialist physician i is under contract j.
h_j	Number of the hours of service by contract j per shift.
de_{smd}	Number of the hours required of skill s per shift m per day d.
ca_j	Cost of the nurse service with contract j per hour.
cb_j	Cost of the general practitioner with contract j per hour.
cc_j	Cost of the specialist physician with contract j per hour.
c_i	Additional service cost per hour for skill i (i = xx, xy, xz).
e	Minimum number of shifts for a full-time care provider.
M	A big number.

Variables

xx_{imd}	One, if nurse i is set for shift m on day d; otherwise, 0.
xy_{imd}	One, if general practitioner i is assigned to shift m on day d; otherwise, 0.
xy_{imd}	One, if specialist i is assigned to shift m on day d; otherwise, 0.
p_{imd}	Number of additional hours required for skill i on shift m per day d.

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$$\text{Min } Z = \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xx}}\sum_{j=1}^{N} h_j ca_j\, aa_{ij} xx_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xy}}\sum_{j=1}^{N} h_j cb_j\, ab_{ij} xy_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xz}}\sum_{j=1}^{N} h_j cc_j\, ac_{ij} xz_{imd} + \sum_{\xi \epsilon B}\sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i \epsilon S} \phi(\xi) c_i\, p_{imd}^{\xi}, \tag{1}$$

$$\sum_{m=1}^{M} xx_{imd} \le 1, \quad i = 1, \dots, I_{xx}, d = 1, \dots, D, \tag{2}$$

$$\sum_{m=1}^{M} xy_{imd} \le 1, \quad i = 1, \dots, I_{xy}, d = 1, \dots, D, \tag{3}$$

$$\sum_{m=1}^{M} xz_{imd} \le 1, \quad i = 1, \dots, I_{xy}, d = 1, \dots, D, \tag{4}$$

$$\sum_{d=1}^{D}\sum_{m=1}^{M} aa_{ij} xx_{imd} \ge e, \quad j = 1, i = 1, \dots, I_{xx}, \tag{5}$$

$$\sum_{d=1}^{D}\sum_{m=1}^{M} ab_{ij} xy_{imd} \ge e, \quad j = 1, i = 1, \dots, I_{xy}, \tag{6}$$

$$p^{\xi}_{xx)md} = \left(de^{\xi}_{xx)md} - \sum_{j=1}^{N}\sum_{i=1}^{I_{xx}} h_j aa_{ij} xx_{imd} \right) f^{\xi}_{xx)md}, \quad m = 1,2, d = 1, \dots, D, \tag{7}$$

$$de_{\,xx)md} \le \sum_{j=1}^{N}\sum_{i=1}^{I_{xx}} h_j aa_{ij} xx_{imd} + M f^{\xi}_{xx)md}, \quad m = 1,2, d = 1, \dots, D, \tag{8}$$

$$de^{\xi}_{xx)md} \ge \sum_{j=1}^{N}\sum_{i=1}^{I_{xx}} h_j aa_{ij} xx_{imd} - \left(1 - f^{\xi}_{xx)md}\right) M, \quad m = 1,2, d = 1, \dots, D, \tag{9}$$

$$p^{\xi}_{xy)md} = \left(de^{\xi}_{xy)md} - \sum_{j=1}^{N}\sum_{i=1}^{I_{xy}} h_j ab_{ij} xy_{imd} \right) f^{\xi}_{xy)md}, \quad m = 1,2, d = 1, \dots, D, \tag{10}$$

$$de^{\xi}_{xy)md} \le \sum_{j=1}^{N}\sum_{i=1}^{I_{xy}} h_j ab_{ij} xy_{imd} + M f^{\xi}_{xy)md}, \quad m = 1,2, d = 1, \dots, D, \tag{11}$$

$$de^{\xi}_{xy)md} \ge \sum_{j=1}^{N}\sum_{i=1}^{I_{xy}} h_j ab_{ij} xy_{imd} - \left(1 - f^{\xi}_{xy)md}\right) M, \quad m = 1,2, d = 1, \dots, D, \tag{12}$$

$$p^{\xi}_{xz)md} = \left(de^{\xi}_{xz)md} - \sum_{j=1}^{N}\sum_{i=1}^{I_{xz}} h_j ab_{ij} xz_{imd} \right) f^{\xi}_{xz)md}, \quad m = 1,2, d = 1, \dots, D, \tag{13}$$

$$de^{\xi}_{xz)md} \le \sum_{j=1}^{N}\sum_{i=1}^{I_{xz}} h_j ab_{ij} xz_{imd} + M f^{\xi}_{xz)md}, \quad m = 1,2, d = 1, \dots, D, \tag{14}$$

$$de^{\xi}_{xz)md} \ge \sum_{j=1}^{N}\sum_{i=1}^{I_{xz}} h_j ab_{ij} xz_{imd} - \left(1 - f^{\xi}_{xz)md}\right) M, \quad m = 1,2, d = 1, \dots, D, \tag{15}$$

$$xx_{imd} \epsilon \{0,1\}, \quad i = 1, \dots, I_{xx}, \; m = 1,2, d = 1, \dots, D, \tag{16}$$

$$xy_{imd} \epsilon \{0,1\}, \quad i = 1, \dots, I_{xy}, \; m = 1,2, d = 1, \dots, D, \tag{17}$$

$$xz_{imd} \epsilon \{0,1\}, \quad i = 1, \dots, I_{xz}, \tag{18}$$

$$m = 1{,}2, d = 1, .., D,$$
$$p_{imd} \geq 0, \quad i \in S, \quad i = xx, xy, xz, \qquad (19)$$
$$m = 1{,}2, d = 1, .., D.$$

The objective function minimizes regular work hours and overtime hours. The first three terms minimize the cost of assigning care providers to work shifts, and the fourth term minimizes the cost of overtime hours caused by higher demand in different scenarios. In this regard, $\phi(\xi)$ is the probability of scenario $\xi = 1{,}2, \dots, B$ and $\sum_{\xi \epsilon B} \phi\ \xi) = 1$. *Constraint (2)* to *Constraint (4)* ensure that each nurse, general practitioner, and specialist physician is assigned to maximally one shift a day. *Constraint (5)* and *Constraint (6)* ensure that the full-time nurses and general practitioners must work on at least e shifts. *Constraint (7)* to *Constraint (9)* specify the number of overtime hours of nursing skills per shift. *Constraint (10)* to *Constraint (12)* specify the number of overtime hours for specialist physicians per shift. *Constraint (13)* to *Constraint (15)* specify the number of overtime hours of nursing skills per shift. Eventually, *Constraint (16)* to *Constraint (19)* define the model's variables.

This research assumes that the required number of hours of skill s on shift m per day (de_{smd}) has a discrete uniform distribution in the interval (a, b). An exact solution can be obtained for small-size problems, but as the size of the problem increases, the solution time increases too. This study solves the problem with the SAA algorithm. A recourse action model is applied to formulate the model of solving the problem with that algorithm. Section 4 delineates the basic features of the new model.

3.1 | Programming with the Stochastic Integer Recourse Model

Stochastic programming models have appeared as extensions of optimization problems with random parameters. Consider the optimization problem below [24]:

$$\begin{aligned} \min \quad & cx, \\ \text{s.t.} \quad & Ax = b, \\ & Tx = h, \\ & x \in X, \end{aligned}$$

where $X \subset \mathbb{R}^n$ indicates the non-negativity of the decision variable x and possibly the integrality constraints on it. In addition to m_1 deterministic constraints of $Ax = b$, there is a set of m constraints of $Tx = h$, where the parameters T and h depend on information and become available only after a decision is made on x. A class of stochastic programming models, known as recourse models, is obtained by allowing additional or recourse decisions after realizing the random variables T and h. So, recourse models are dynamic; the stages model the time discretely based on the existing data. If the uncertainty is all dissolved simultaneously, it can be captured by a recourse model in two stages, present, and future. Considering a first-stage decision x, the infeasibility of $h - Tx$ for every possible (q, T, h) is compensated with minimum costs. In contrast, second-stage decisions are made as an optimal solution to the second-stage problem. This specifies the minimal recourse costs as a function of the first-stage decision x, and the realization of ξ is denoted by $\upsilon(x, \xi)$. Its expectation, $Q\ x) = \mathbb{E}_{\xi}[\upsilon\ x, \xi)]$, yields the expected recourse costs associated with the first-stage decision x. Thus, the two-stage recourse model is:

$$\begin{aligned} \min \quad & cx + Q\ x), \\ \text{s.t.} \quad & Ax = b, \\ & x \in X, \end{aligned}$$

where the objective function $cx + Q\ x)$ specifies the total expected costs of decision x [24]. The stochastic demand model of scheduling care providers can be expressed as the following recourse model.

$$\text{Min Z} = \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xx}}\sum_{j=1}^{N} h_j ca_j\, aa_{ij} xx_{imd}$$
$$+ \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xy}}\sum_{j=1}^{N} h_j cb_j\, ab_{ij} xy_{imd}$$
$$+ \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xz}}\sum_{j=1}^{N} h_j cc_j\, ac_{ij} xz_{imd} + E[Q\ x,\xi)],$$

$$s.t.\ \ 1) - \ 5),$$

where $E[Q(x,\xi)]$ is the recourse action function, and

$$Q\ x,\xi) = \min \sum_{\xi\epsilon B}\sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i\epsilon S} \phi(\xi) c_i\, p_{imd}^{\xi},$$

$$\text{s.t.}\ \ 6) - \ 16).$$

The $\xi \in B$ vector contains numerous scenarios. So, to obtain $E[Q\ x,\xi)]$, lots of similar Integer Linear Programs (ILPs) [25] must be solved, which is a difficult calculation task. Since it is hard to provide an exact solution to the proposed model, the following section proposes an approximation.

4 | Sample Average Approximation

Several solution methods, such as SAA, exist to solve stochastic models. The SAA method is a Monte Carlo simulation-based method that solves stochastic programming problems by generating random samples and approximating the expected function values through the average functions of the corresponding samples. The stop criterion determines how long the algorithm will last. Over the years, various authors have used the idea of SAA to solve stochastic programs. For example, it was employed to solve stochastic knapsack problems [26], stochastic routing problems [27], supply chain problems [28], and investment problems [29]. Due to the high applicability of the SAA method, it has been selected to solve the model in this study. The method is delineated below:

Suppose M is the number of replications, N is the number of scenarios in the sample problem, and N' denotes the sample size used to estimate $C^T\widehat{X} + E[Q(\widehat{X},\xi)]$ for a given feasible solution $\widehat{X}$. So, the SAA method can be described as follows [27]:

I. Repeat the following steps for m = 1, ..., M:

Generate $\xi^1, \xi^2, \ldots, \xi^N$ as an N-random sample

Solve the problem by the SAA method and take $\widehat{X}_N^m$ as a solution vector and $\widehat{Z}_N^m$ as an optimal objective value.

Generate $\xi^1, \xi^2, \ldots, \xi^{N'}$ as an independent random sample. Evaluate $\widehat{g}_{N'}(\widehat{X}_N^m)$ and $S^2_{\widehat{g}_{N'}(\widehat{X}_N^m)}$ as follows:

$$\hat{g}_{N'}\left(\hat{X}_N^m\right) = \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xx}}\sum_{j=1}^{N} h_j ca_j\, aa_{ij} xx_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xy}}\sum_{j=1}^{N} h_j cb_j\, ab_{ij} xy_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xz}}\sum_{j=1}^{N} h_j cc_j\, ac_{ij} xz_{imd} + \frac{1}{N'}\sum_{n=1}^{N'}\sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i\epsilon S} \phi(\xi) c_i\, p_{imd}^{\xi}$$

$$S^2_{\hat{g}_{N'}(\hat{X}_N^m)} = \frac{1}{N'(N'-1)}\sum_{n=1}^{N'}\left[\sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xx}}\sum_{j=1}^{N} h_j ca_j\, aa_{ij} xx_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xy}}\sum_{j=1}^{N} h_j cb_j\, ab_{ij} xy_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I_{xz}}\sum_{j=1}^{N} h_j cc_j\, ac_{ij} xz_{imd} + \sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{i\epsilon S} \phi\ \xi) c_i\, p_{imd} - \hat{g}_{N'}(\bar{X})\right].$$

$$\bar{Z}_N^M = \frac{1}{M}\sum_{m=1}^{M} \hat{Z}_N^m\ , S^2_{\bar{Z}_N^M} = \frac{1}{M(M-1)}\sum_{m=1}^{M}[\hat{Z}_N^m - \bar{Z}_N^M].$$

II. Evaluate $\bar{Z}_N^M$ and $S^2_{\bar{Z}_N^M}$.

The following formula serves to calculate the confidence interval for the optimality gap:

$$\bar{Z}_N^M = \frac{1}{M}\sum_{m=1}^{M} \hat{Z}_N^m\ , S^2_{\bar{Z}_N^M} = \frac{1}{M(M-1)}\sum_{m=1}^{M}[\hat{Z}_N^m - \bar{Z}_N^M].$$

$$\hat{g}_{N'}\left(\hat{X}_N^m\right) - \bar{Z}_N^M + Z_\alpha\left\{S^2_{\hat{g}_{N'},\ \bar{X})} + S^2_{\bar{Z}_N^M}\right\}^{0.5}.$$

Here is $Z_\alpha = \Phi^{-1}(1-\alpha)$, in which $\Phi(Z)$ stands for the cumulative pattern of the standard normal distribution.

III. In the case of each solution $\hat{X}_N^m$, the parameter m = 1, ..., M determines the optimality gap with $\hat{g}_{N'}\left(\hat{X}_N^m\right) - \bar{Z}_N^M$ along with an estimated variance of $S^2_{\hat{g}_{N'}(\hat{X}_N^m)} + S^2_{\bar{Z}_N^M}$. One of the M candidate solutions is selected based on the least estimated objective value.

In the algorithm, $\bar{Z}_N^M$ and $\hat{g}_{N'}\left(\hat{X}_N^m\right)$ are the lower and upper bounds of the optimal value, respectively [30]. The parameter $\bar{Z}_N^M$ shows an unbiased estimator of the optimal objective function $E(\hat{Z}_N)$. Here, $\bar{Z}_N^M = E(\hat{Z}_N)$ and $E(\hat{Z}_N) \le Z^*$. Moreover, $\hat{g}_{N'}\left(\hat{X}_N^m\right)$ presents an unbiased estimator of the objective value $E(\hat{Z}_N)$, but $E(\hat{g}_{N'}\left(\hat{X}_N^m\right)) \ge Z^*$.

5 | Numerical Results

5.1 | Case Study

This section reports a case study planned at the Iranian Health Control Center. There were nine nurses, seven general practitioners, and three specialist physicians with 24 working days divided into two shifts, morning and afternoon. The corresponding data were obtained from the Iranian Health Control Center to evaluate the distribution of the demand for each skill each day and each shift. The demand had a uniform distribution in the intervals (24, 36), (12, 18), and (5, 9) per hour for the nurse, general practitioner, and specialist physician in each shift, respectively. *Table 3* shows the cost per contact hour for different skills. The cost of each additional hour of service for a nurse, general practitioner, and specialist physician is 90, 160, and 240 dollars, respectively. Contracts are available in three types, full-time, part-time, and hourly, containing 8 hours, 4 hours, and 2 hours per shift, respectively. The minimum number of shifts(e) for full-time contract nurses and general practitioners is 20, which is obtained based on the information from the Iranian Health Control Center. *Table 4* presents the summary of the parameters.

Table 3. The wage of each skill per hour ($).

	Nurse	General Practitioner	Specialist Physician
Full time	50	60	110
Part-time	60	110	150
Hourly	70	150	200
Cost of an additional hour	90	160	240

Table 1. Summery of parameters.

Contracts	Full-Time, Part-Time, Hourly
Skills	Nurse, general practitioner, specialist physician
$h_{full-time}$	8
$h_{part-time}$	4
h_{hourly}	2
e	20

The proposed approach was put to practice in Python, employing a GUROBI optimization solver (http://www.gurobi.com/) on a mac book pro with an 8-core CPU and 8-GB RAM.

5.2 | Numerical Results

This section is dedicated to the experimental results of implementing our approach in the Iranian Health Control Center. In this approach, the N quantities of 1, 20, 50, and 100 and the M quantities of 10 and $N' = 20000$ for the SAA algorithm presented in Section 3, and the Monte Carlo method was used to generate random numbers. *Table 5* to *Table 8* show the results. Column $\widehat{g}_{N'}(\widehat{X})$ shows the value of the objective function based on N' independent random sample, and column $S^2_{\widehat{g}_{N'}(\widehat{X})}$ indicates the value of variance. Column $\widehat{Z}_N^m$ specifies the optimal value of the objective function considering the N scenario. The gap column shows $\widehat{g}_{N'}(\widehat{X}) - \widehat{Z}_N^m$, and Var column refers to $S^2_{\widehat{g}_{N'}(\widehat{X})} + S^2_{\widehat{Z}_N^m}$. As the value of N increased from 1 to 20, 50, and 100, the Var changed from 222039.63 to 56973.116, 10280.988, and 3449.325; there was a decrease of 99.97% in total. Moreover, with the increase of N, the mean values of $\widehat{g}_{N'}(\widehat{X})$ and $\widehat{Z}_N^m$ began to converge to make an optimal solution. The mean of $\sum_{i=1}^{M} \widehat{g}^i_{N'}(\widehat{X})/M - \overline{Z}_N^M$ decreased from 10976.77 to 251.16 for N values of 1 to 100. The total reduction was 97.8%. These values showed a convergence for an optimal solution.

Table 5. Simple Monte Carlo method: N = 1, M = 10 and N' = 20000.

m	$\hat{g}_{N'}(\hat{X})$	$S^2_{\hat{g}_{N'}(\hat{X})}$	$\hat{Z}^m_N$	Gap	Var
1	198972.3	286.3	184279	13881.35	222060.9
2	195957.3	263.64	185240	10866.42	222038.2
3	195868.2	297.41	185570	10777.28	222072
4	197264	243.38	188200	12173.06	222018
5	196737.7	253.38	183060	11646.75	222028
6	195290.8	259.71	184470	10199.94	222034.3
7	193692.3	246.14	185130	8601.37	222020.7
8	195807.1	268.63	186560	10716.22	222043.2
9	196799.7	274.34	183450	11708.76	222048.9
10	194287.3	257.48	184950	9196.35	222032.1
$\bar{Z}^m_N = 185090.9$					
$S^2_{\hat{Z}^m_N} = 221774.59$					

Table 6. Simple Monte Carlo method: N = 20, M = 10 and N' = 2000.

	$\hat{g}_{N'}(\hat{X})$	$S^2_{\hat{g}_{N'}(\hat{X})}$	$\hat{Z}^m_N$	Gap	Var
1	189620.6	260.45	188152.5	953.4	56977.13
2	189600.5	257.26	189520	933.273	56973.94
3	189591.9	255.13	188311.5	924.7	56971.81
4	189544	260.26	187329.5	876.76	56976.94
5	189592	258.66	188913.5	924.83	56975.34
6	189613.1	259.02	189344.5	945.89	56975.7
7	189583.7	258.92	187789.5	916.47	56975.6
8	189544.2	253.49	188928	877.02	56970.17
9	189689.9	252.14	189571.5	1022.66	56968.82
10	189642.1	249.03	188811.5	974.85	56965.71
$\bar{Z}^m_N = 188667.2$					
$S^2_{\hat{Z}^m_N} = 56716.68$					

Table7. Simple Monte Carlo method: N = 50, M = 10 and N' = 20000.

	$\hat{g}_{N'}(\hat{X})$	$S^2_{\hat{g}_{N'}(\hat{X})}$	$\hat{Z}^m_N$	Gap	Var
1	189523.9	258.39	189129.4	515.29	10281.55
2	189554.4	257.36	189046.4	545.8	10280.52
3	189502.4	263.52	188939.8	493.75	10286.68
4	189510.6	254.97	188681.8	501.98	10278.13
5	189529.3	257.63	188575	520.65	10280.79
6	189529.2	259.21	189338.8	520.61	10282.37
7	189543.1	254.34	188856.2	534.51	10277.5
8	189519.1	259.17	189368	510.43	10282.33
9	189488.9	258.57	188677.8	480.26	10281.73
10	189546.5	255.12	189473	537.86	10278.28
$\bar{Z}^m_N = 189008.62$					
$S^2_{\hat{Z}^m_N} = 10023.16$					

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Table 8. Simple Monte Carlo method: N = 100, M = 10 and N' = 20000.

	$\widehat{g}_{N'}(\widehat{X})$	$S^2_{\widehat{g}_{N'}(\widehat{X})}$	$\widehat{Z}_N^m$	Gap	Var
1	189520.5	261.47	189356.8	243.11	3454.56
2	189500.7	255.04	189063.1	223.32	3448.13
3	189524.1	252.5	189333.3	246.72	3445.59
4	189565.9	254.14	189285.6	288.46	3447.23
5	189500.6	258.71	189063.4	223.16	3451.8
6	189551.9	251.47	188997.8	274.48	3444.56
7	189526.2	256.26	189471.2	248.76	3449.35
8	189527	257.76	189517.6	249.65	3450.85
9	189543.4	258.78	189388.6	266.04	3451.87
10	189525.2	256.22	189296.5	247.8	3449.31
$\bar{Z}_N^m = 189277.39$					
$S^2_{\bar{Z}_N^m} = 3193.09$					

The next step generated uncertain parameters with the Latin Hypercube Sampling method [31] to reduce the SAA method's variance. This is a stratified random sampling method by which samples are selected from many variables so that the sample for each variable has the highest degree of classification. As shown in *Table 9*, with the Latin Hypercube Sampling method, the variance at the lowest point (N = 100) in the Monte Carlo simulation decreased from 3449.325 to 141.531, which means a reduction of 96%. *Fig. 1* and *Fig. 2* show the box plot for the Monte Carlo sampling method and the Latin Hypercube Sampling method, respectively. As can be seen, the dispersion of the values of $\widehat{g}_{N'}(\widehat{X})$ and $\widehat{Z}_N^m$ obtained in the Monte Carlo sampling method is greater than the Latin Hypercube Sampling method.

Table 9. LHS method (N = 100, M = 10 and N' = 20000).

	$\widehat{g}_{N'}(\widehat{X})$	$S^2_{\widehat{g}_{N'}(\widehat{X})}$	$\widehat{Z}_N^m$	Gap	Var
1	189582.3	14.17	189340.8	273.01	143.57
2	189577.4	14.36	189308.1	268.12	143.76
3	189582.2	14.07	189332.8	272.93	143.47
4	189580.7	14.09	189281.3	271.4	143.49
5	189578.6	13.89	189365.4	269.33	143.29
6	189576.2	14.01	189288.6	266.88	143.41
7	189578.7	14.31	189247.3	269.45	143.71
8	189582.9	14.3	189275	273.61	143.7
9	189579	13.96	189319.3	269.68	143.36
10	189577.5	14.15	189334.2	268.18	143.55
$\bar{Z}_N^m = 189309.28$					
$S^2_{\bar{Z}_N^m} = 129.4$					

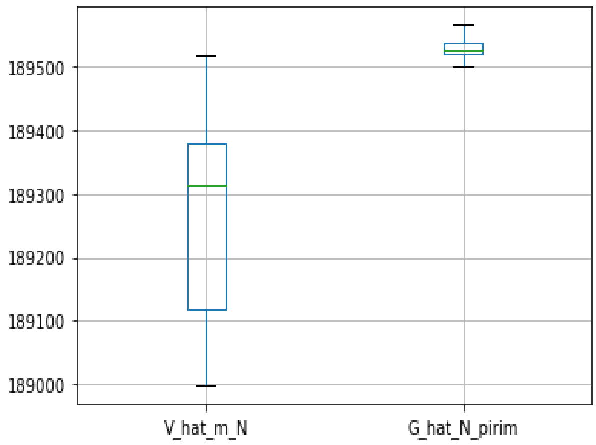

Fig. 1. Simple Monte Carlo method (N = 100, M = 10 and N' = 20000).

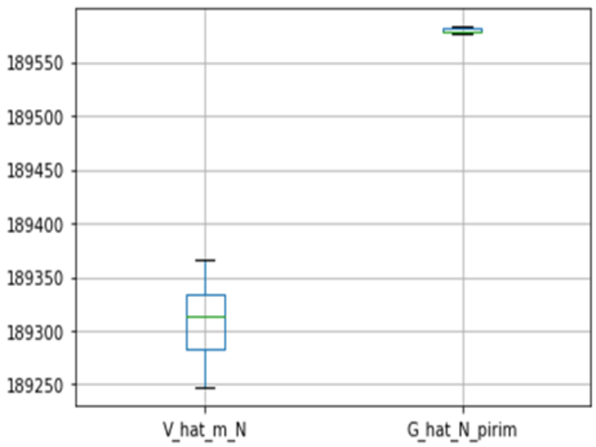

Fig. 2. LHS method (N = 100, M = 10 and N' = 20000).

Any significant difference between the means of the gap of Monte Carlo and the Latin Hypercube Sampling methods was checked with the Mann-Whitney test. As the test was performed with $\alpha = 0.05$, the null hypothesis was rejected (p-value = 0.013). Therefore, a significant difference was detected between the means of those two methods. Since the Latin Hypercube Sampling method reduced the variance (Var) to 96%, the least value of the objective function of this method was selected. Based on the results, 189247.3 was the lowest objective function value and, thus, was selected as the best answer. *Table 10* to *Table 12* show the shift plans of nurses, general practitioners, and specialist physicians determined based on the best response. In these tables, morning or afternoon shifts are assigned to each care provider; otherwise, that provider would not be busy. The letters M and A on the tables stand for the morning and afternoon shifts, respectively. According to the obtained schedule, nurses' workload is more than other care providers due to the higher demand for this skill. In the case of general practitioners and specialist physicians, the workload is less; however, the supply of human resources is more complex, and proper planning should be done for this matter.

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Table 102. Nurses' work schedule.

	Days 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	A	M	M	M	A	A	A	A	A	M	A	A	A	A	A	A	A	A	M	M	A	M	M	A
2	M	M	M	M	M	A	A	A	M	M	M	A	M	A	A	M	A	M	M	A	M	A	M	A
3	M	M	A	M	M	M	M	M	M	A	M	M	A	M	M	M	A	M	A	M	M	M	A	A
4	A	A	M	A	A	M	M	M	M	A	A	A	A	M	M	A	M	A	M	A	A	A	M	M
5	A	A	A	A	A	A	M	A	A	A	M	M	M	A	M	M	M	A	A	M	A	M	A	M
6	M	A	A	A	M	A	A	M	A	A	A	M	M	M	A	A	M	M	A	A	M	M	A	M
7	A	A	A	A	M	M	M	A	A	M	A	M	M	A	A	A	M	A	A	A	M	A	A	M
8	M	A	A	A	A	M	A	M	A	M	M	M	M	M	M	M	M	M	A	M	A	A	A	M
9	M	A	M	A	M	A	M	A	A	M	M	A	A	A	M	A	M	M	M	M	A	A	M	M

Morning shift   Afternoon shift

Table 11. General practitioners' work schedule.

	Days 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	A	M	M	A	M	A	A	A	A	A	A	A	A	A	A	A	A	M	M	M	A	A	M	A
2	M	A	A	M	A	M	M	M	M	M	M	M	M	M	M	M	M	A	A	A	M	M	M	M
3	A	M	A	A	A	A	M	A	A	M	M	A	A	M	A	A	M	A	M	A	A	M	A	A
4	-	-	-	-	-	-	-	-	-	A	-	-	M	A	M	M	A	-	-	M	-	A	A	M
5	-	A	M	-	-	M	A	M	M	-	-	-	-	-	-	-	-	-	A	-	M	-	-	-
6	M	-	-	M	M	-	-	-	-	-	A	M	-	-	-	-	-	M	-	-	-	-	-	-
7	M	A	M	M	M	M	A	M	M	A	A	M	M	A	M	M	A	M	A	M	M	A	A	M

Table 12. Specialist physicians' work schedule.

	Days 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	M	M	M	M	-	M	-	M	M	-	M	M	M	-	M	A	M	M	M	M	-	-	M	M
2	A	A	A	A	M	A	M	A	A	M	A	A	A	M	A	-	A	A	A	A	M	A	A	A
3	-	-	-	-	A	-	A	-	-	A	-	-	-	A	-	M	-	-	-	-	A	M	-	-

After the service planning was conducted, the Monte Carlo simulation method validated the model. As many as 1000 problems were generated randomly, and calculations were performed to obtain the values of their objective functions. The histogram in *Fig. 3* presents the obtained values. As the results revealed, the average value of the total cost was 191521, which shows a 1% difference from the best solution obtained through solving the model by the SAA method. The worst value of the objective function obtained from the simulation method was 1974480, which shows a 4% difference from the best result obtained by solving the SAA model.

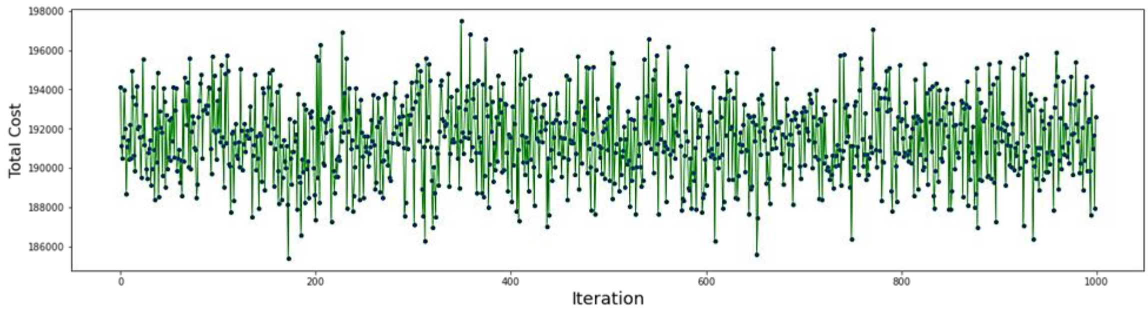

Fig. 1. Simulation results.

5.3 | Managerial Implications

In addition to the significant cost reduction resulting from more efficient shift scheduling, the daily use of shift schedules has important managerial implications for the workload of home care, hospital administrators, and care providers. This schedule can free the care providers to deal with tasks requiring closer patient interactions. Moreover, setting a shift schedule makes it possible to hold training courses and update the care providers in their free time. Another advantage of this planning is to provide a robust program against changes in patient demand. The continuous use of planning can be beneficial for patients. If they ever face a shortage of care providers, the necessary predictions have already been made in a schedule. To provide better plans, in this case, continual cooperation between the Iranian Health Control Center and universities seems necessary. Hospitals and other home care centers can also plan with more constraints if required.

6 | Conclusion

In real-world shift scheduling, care providers cooperate with various practitioners and work by different contracts in health care centers under different sources of uncertainty, such as patient demand. Among the patients for whom demand uncertainty may occur are cancer patients, who experience unpredictable conditions during their illness. This study addresses the issue, and the demand distribution is estimated based on the available data. To this end, a two-stage stochastic programming model is presented, and the problem is solved with the SAA method. Based on the results, by increasing the sample size (N) from 1 to 100, the upper and lower bounds of the optimal solution begin to converge to the optimal solution. Monte Carlo and Latin Hypercube Sampling methods were used for sampling in the SAA method. The results show that the Latin Hypercube Sampling method has 96% less variance than the Monte Carlo method. Also, the distance between the upper and lower bounds for Latin Hypercube Sampling is 0.143%, which shows the proximity of the solution to the optimal value. Then, based on the best answer obtained from the Latin Hypercube Sampling method, the work schedule of care providers is presented in timetables. Finally, the solution method has proved to be efficient by using the Monte Carlo simulation method.

There are several recommendations to make for future research in nurse scheduling. The specific skills needed by individual patients can be considered a basis for assigning relevant care providers. Moreover, human factors involved in care providing seem exciting topics to study. Other methods can also be tried to solve stochastic programming models. Also, different robust approaches can be used to face the uncertainty of patients' demands. Finally, models may be developed in areas like fire stations and emergency centers where shift planning is needed.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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The Impact of Discount Policy on Hotel Pricing in a Competitive Market after Covid-19 Using Game Theory

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Abstract

This paper presents an optimization model for hotel pricing in the competitive environment following the Covid-19 epidemic, in which the government intervenes by offering appropriate tariffs and hotels use incentive policies such as discounts to attract customers. we consider the government as the leader and the hotels as the followers of the Stalkberg model, then apply the Nash equilibrium to determine the optimal price and demand of hotels in competitive conditions, taking into account the discount. By considering a government utility function, the optimal level of government tariffs is determined. The results indicate that government intervention in the tourism industry includes measures that benefit tourism. Because the government can increase the hotel revenue and expand tourism in favor of hoteliers by reducing its profits. Extensive analysis has been performed on five-star, four-star, and three-star hotels in a tourist area in Iran, and some of the most important managerial insights have been explained.

Keywords: Hotel pricing, Discount, Game theory, Tourism industry, Stakelberg game.

1 | Introduction

The tourism industry, one of the broadest development prospects of the 21st century, has the potential for sustainable development and risk resistance [1]. In the last two decades, tourism has experienced a continuous and profound expansion and has become one of the fastest economic sectors in the world, becoming the fourth largest export industry after the oil, petrochemical, and food industries [2], [3]. For this reason, the tourism industry is considered as a vital tool for the development and improvement of economic conditions in many parts of the world [4]. Despite all the capabilities of the tourism industry, it is highly vulnerable to external disasters such as natural disasters such as floods, earthquakes, hurricanes, terrorism, and fires, and the spread of epidemics such as SARS and, more recently, the Corona pandemic. Therefore, any disasters such as the spread of an epidemic or the start of a war, or other natural disasters will lead to a recession in the industry and cause serious damage to it [4], [5]. After several people contracted unusual pneumonia in December 2019, China

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introduced a new strain of the coronavirus as the cause of a new respiratory illness. With the rapid spread of the disease in China and then to other parts of the world, the new coronavirus, scientifically known as acute coronavirus syndrome (SARS-Cov-2), and the resulting disease, known as Quaid-19 disease, has caused great concern and panic among the people of the world [6]. Covid-19 has caused serious and irreparable damage to the tourism industry and posed serious challenges to all related sectors. According to figures released by the World Bank, the Covid-19 epidemic has plunged the world economy into its worst recession since World War II, with a 4.3% drop in global GDP by 2020 [7]. Among all the Covid-19-influenced industries, tourism was among the most affected, with an epidemic that reduced international tourism inflows by 74% in 2020 compared to 2019. In addition to concerns about the spread of the Corona pandemic, the growing increase in human activity has produced large amounts of Greenhouses Gas (GHG), leading to severe global warming and climate change. Many researchers believe that the current global warming scenario, particularly GHG emissions are driven by human activities, by energy-intensive sectors such as hotels [8]. To maintain sustainable development, governments generally implement various measures and policies to reduce emissions [9]. Because green tourism can be described as a concept similar to sustainable development. Thus, green tourism embraces the concept of sustainable development and is based on the idea of harmony between people and the environment. Therefore, green tourism means that tourism activists, including hoteliers, tourists, restaurants, travel agencies, and tour guides, must respect nature and protect the environment in all aspects of the entire tourism process [1]. To this end, hotels as one of the most important factors in the development of tourism in countries adopt new approaches to make optimal decisions for the growth and development of the tourism industry [10]. To achieve this goal, hoteliers rely on adopting an essential strategy to compete and gain a competitive advantage [11]. In fact, this particular approach can lead hoteliers to more stable incomes than competitors [12]. Based on the above, the widespread prevalence of pandemics such as Covid-19, wars, and natural and unnatural disasters can have a detrimental effect on tourist attractions. In this study, we intend to investigate the effect of discounts on prices as a balanced competitive strategy for the post-pandemic period in the tourism industry. Because, we believe that the adoption of discount policies can deal with the damage caused by reducing the attractiveness of the tourism industry in unusual conditions, and in addition, hoteliers also enjoy the economic benefits of expanding the tourism industry in a challenging competitive environment with which the world today Is involved in maintaining their income. The purpose of this study is to provide a suitable model for discount-based pricing for hotels that compete with the same stars (three stars, four stars, five stars) but different services. The purpose of this study is to provide a suitable model for discount-based pricing for hotels that compete with the same stars (three stars, four stars, five stars) but different services. The most important contribution of this article is to consider the incentive policy of the government as a leader and the discount policy of hotels as a follower.

- *What effect do incentive policies to attract tourists, such as discounts or the use of free facilities in tourism complexes, have on customer demand in post-coronary conditions?*
- *What is the role of the government as a leader in the post-corona situation to support hoteliers?*
- *What is the impact of prices on tourist demand?*

The rest of this article is organized as it turns out. In Section 2, a literature review of the main research topic is presented. In Section 3, the proposed method for research is introduced. In this section, while introducing the problem in detail, the relevant modeling is stated. In Section 4, the results of using the proposed model are shown. Finally, the Section 5 provides an overview with suggestions for future research.

2 | Literature Review

Covid-19 is an unknown disease that the world has been affected, therefore as a result of this crisis the hotel industry is facing severe challenges [13]. This major challenge has affected all stakeholders in the hotel industry due to travel bans and social distance laws, and as a result, tourists' willingness and access to travel have decreased [14]. This has led tourists to cancel travel and hotel reservation plans, which

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has ultimately affected job security and the income of hotel staff [15]. In this situation, many hotel owners have temporarily closed or transferred their property, in which case this scenario has led to a sharp decline in the stock market value in the hotel sector. In other words, the epidemic has devastated the market and the performance of hotel companies [13]. In this regard, Chopra et al. [16] implemented different pricing strategies for companies in the face of the Quidd-19 pandemic crisis. To this end, the pricing strategies of several high-risk industries were discussed. This study helps companies understand the economic situation and develop pricing strategies based on it. Such pricing strategies allow companies to satisfy their customers in order to stay competitive and achieve business superiority. In general, based on studies, we classify the research literature into three categories; 1) revenue management in the hotel industry, 2) pricing in the hotel industry, and 3) discount pricing models in the hotel industry. The first category is revenue management, which is defined as an information and pricing system to achieve sufficient capacity at the right time and in the right place and thus maximize revenue [17], [18]. To this end, Aydin and Birbil [19], using the dynamic analysis method, provided a framework for managing hotel revenue in room allocation. Paving the way for future hotel revenue management studies on late payment and overbooking is one of the upstream goals in this study. To this end, several room allocation policies in hotel revenue management have been examined. Baker et al. [20] studied hotel revenue management with the aim of organizing the literature on hotel revenue management systems and comparing hotel management systems in terms of performance. In this study, new areas of forecasting include creating a measure of the degree of appropriateness of a hotel pricing strategy and using it to quantify online surveys for forecasts. New ways of optimizing prices, including determining whether a mismatch between a customer's perception of justice and trust and pricing history have a detrimental effect on overall hotel performance, have been introduced as new pricing methods. Klein et al. [21] provide an overview of revenue management and its applications in various industries such as aerospace, hospitality, delivery, and manufacturing since 2004. The second category is determining the optimal price for hotels, which is a complex process. Because, hotels have to consider various factors such as accessibility, customer evaluation of the services provided and future demands [22]. The important point in determining the optimal price is the optimal allocation [23]. Vives and Jacob [24] presented a model for dynamic pricing for hotel demand. The results of this study show that 1) seasonality, number of available rooms, hotel location, and tourist characteristics affect dynamic pricing, 2) reservation constraints lead to further reductions in revenue under elastic demand, 3) higher levels of demand elasticity usually produce lower levels of prices, and 4) distribution of elasticities across the horizon of reserve and natural variability of demand affects dynamic pricing. Mariello et al. [25] provide a simulation optimization-based framework for hotel pricing. In this paper, we introduce hotel simulation as an optimization approach based on flexible simulation. They used parametric demand models to inject new information into the simulator and adapt pricing policies to mutant market conditions. Also, cancellations and reservations are made on an intermediate basis and seasonal averages can be adjusted on a daily basis. Mousavi et al. [26] presented a model for optimizing prices in a competitive environment from the perspective of energy-saving and environmental protection, in which the government intervenes by providing appropriate tariffs for their performance. Extensive analysis was performed on hotels in a tourist area in Iran and some of the most important managerial insights are explained. This study, for the first time, examines the impact of government interference on hotel pricing as hotels compete with different characteristics. Hence, a decentralized decision-making structure is considered for hotels. In addition, this paper presents a new model for providing optimal prices for hotels in a competitive market, optimal hotel revenues, and optimal government tariffs. The third category is discount pricing models in times of crisis and when demand is declining unusually. For example, Kim et al. [27], in their paper, analyze the effect of price reductions on improving hotel performance. Given that crisis management strategies are very important for hotels; little research has been done. As a pioneer in this regard, this study examined the effect of price on improving the performance of hotels and showed that price reduction helps hotel management. Lotfi et al [28] proposed a novel viable a Medical Waste Chain Network Design (MWCND) by a novel two-stage robust stochastic programming that considers resiliency (flexibility and network complexity) and sustainable (energy and environment) requirements. Lotfi et al [29] explored a Robust, Risk-aware, Resilient, and Sustainable Closed-Loop Supply Chain Network Design (SCND) (3RSCLSCND) to tackle demand fluctuation like Covid-19 pandemic. For this purpose, a two-stage robust stochastic multiobjective programming model serves to express the proposed problems in formulae. Lotfi et al. [30] indicated resilience and sustainable SCND by considering Renewable Energy (RE) for the first time. A two-stage new robust stochastic optimization is embedded for RSSCNDRE. The first stage locates facility location and RE and the second stage defines flow quantity between Supply chain components. Based on the above, research that can provide a discount pricing model for the hotel industry based on the knowledge gained is less visible. Therefore, in this study, we intend to provide a suitable model for the price and demand of hotels in a tourist area by considering the discount coefficient to encourage tourists when they desire to stay and attract tourists decreases.

3 | Problem Description and Model Presentation

3.1 | Prerequisites and Assumption

It is assumed that hotels i and j with the same stars, which are green and non-green, are competing with each other in a tourist area in northern Iran. Accordingly, hotel i strives to provide better green services compared to hotel j in terms of hotel management indicators. The hotel index is a criterion that is determined based on the pollution standards of each country. The higher this value means that the hotel is performing well in its public uses. These hotels lost a lot of demand in the face of the Covid-19 pandemic outbreak, but are now competing with each other to attract tourists by reducing their initial inflammation by pursuing their own policies. The policy of green hotel is to rely on the environment friendly aspect, and the policy of non-green hotels is to emphasize discounts on prices. Given the above, in this study, we want to examine the impact of each of the above hypotheses on demand. The symbols used in this research are shown in *Table 1*.

Table 1. Model symbols.

Symbol	Description
p'_i	Hotel prices i taking into account government tariffs
d	Replacement coefficient of two hotels with each other
p'_j	Hotel prices j taking into account government tariffs
β	Sensitivity of a hotel's demand to the level of competitor service
s_i	Hotel index for hotel i
y	Sensitivity of a hotel's demand to the level of competitor service $\beta > y$
s_j	Hotel index for hotel j
c_i	Variable cost for hotel i
c_j	Variable cost for hotel j
η_i	Hotel service investment efficiency coefficient i
η_j	Hotel service investment efficiency coefficient j
c'_i	Fixed fee for hotel i
c'_j	Fixed fee for hotel j
μ	Modulator of tourist attraction for hotel i in competition with hotel j; $0 < \mu < 1$
γ_i	Price adjustment rate for hotel i at a discount
γ_j	Price adjustment rate for hotel j in terms of discount
ϕ	Energy saving coefficient for hotel demand base i in competition with hotel j; $0.5 < \phi < 1$
a	Random base market value for hotels
Ω	Relative weight ratio between tourism industry expansion and hotel demand
λ	The relative weight of government revenue relative to hotel revenue
U	Government utility function

In this study, we consider two distinct hotels with the same star in a tourist area that has many hotels. Although the hotels are the same stars, the hotel service index in them is different and as a result, this affects the price of hotels. Waste management, energy-saving is among the policies included in the hotel index. The study uses the Stackelberg game with decentralized conditions, in which the government, as a leader, oversees the performance of hotels and can impose various tariffs on hotels at its discretion. For example, the equilibrium rate for hotels i and j is set by the government. In the Stalkberg game, the leader is the provider of better goods and services, which is in a higher position than the follower and has more power. However, after observing and considering the market leader's decision, the follower determines his strategy in a game. In this case, if the government decides to boost tourism and encourage hotels to make better use of resources by subsidizing hotels, then prices will be affordable for customers. In contrast, in crowded tourist areas, the government can make money by imposing significant taxes on hotels. Also, when demand falls sharply; for example, when faced with a corona pandemic, the government imposes a percentage discount on hoteliers to consider a discount on their price. The most important goal in this study is to analyze the interactions between hotels at a time when the government is interfering in their performance. In addition, these hotels can have a centralized or decentralized decision-making structure. In this regard, we assume that hotels compete with each other, although each can set its own prices. Of course, the price is set by the hotels using the discount rate set by the government based on the type of hotel. *Fig. 1* schematically shows the competitive orientation of hotels in a decentralized decision-making structure.

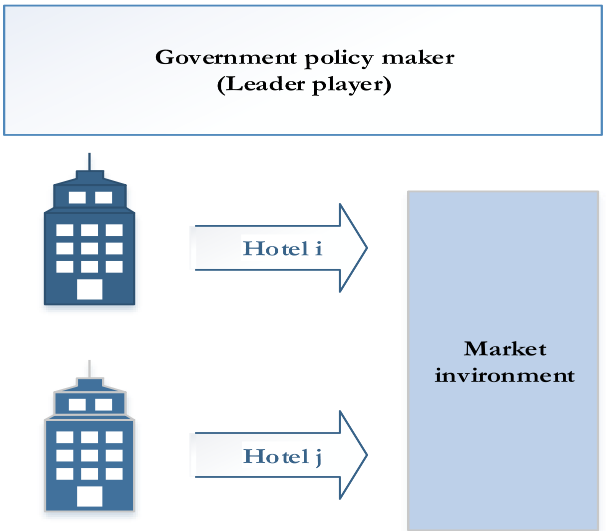

Fig. 1. Government intervention in competition with hotels.

3.2 | Research Modeling

The development of the model presented in this paper is in line with the development of the model of Mousavi et al. [26], by adding the idea of discount as follows. In the proposed model, it is assumed that the discount in hotel i or j is offered when demand decreases, in which case the hotel will offer a discount of γat its price $P'_{I,j}$. In the proposed relations, γis considered as a parameter whose value is determined by the government. Therefore, if the price after the value added is equal to $P'_{i,j} = \left(P_{i,j} + t\right)$ after applying the discount percentage as $P'_{i,j} = \gamma_{i,j}\left(P_{i,j} + t\right)$ will be converted. In fact, γ plays the role of a discount rate for the price of hotel accommodation. For this purpose, for the amount of demand of each hotel i or j, we consider two amounts of demand as a set of *Eqs. (1)* and *(2)*. So that in one relation the mode with discount is considered and in the other relation the mode without discount is considered. It should be noted that both relationships are not established and in each case only one relationship is established. For this purpose, we consider a binary variable that plays such a role.

$$\tilde{q}_I = f\mu a - \gamma_I P'_I + d\gamma_I P'_j + \beta s_I - y s_j + MK_1. \quad (1\text{-a})$$

$$\tilde{q}_I = f\mu a - P'_I + dP'_j + \beta s_I - y s_j + MK_2. \quad (1\text{-b})$$

$$K_1 + K_2 = 1.$$

$$K_1, K_2 \in \{0,1\}.$$

Inset *Relation (1)*, if:

$K_1 = 0$, demand is low, discount model is applied for hotel i,

$K_2 = 1$, since the value of M is a very large number, the constructed relation is considered as a redundant relation whose scale is not considered in the dimensions of the problem.

Therefore, taking into account the above assumptions using *Eq. (1-b)*, the demand model is created by considering the price discount and *Eq. (1-b)* is discarded. In this way, a discount model is applied, which is examined in this article. We also examine relationships related to the amount of hotel demand j under the terms of the discount. In the set of *Relation (2)*, like hotel i, we examine the model of hotel demand j under discount conditions.

$$\tilde{q}_J = (1-\mu) a - \gamma_j P'_j + d\gamma_j P'_I + \beta s_j - y s_I + MK_3. \quad (2\text{-a})$$

$$\tilde{q}_J = (1-\mu) a - P'_I + dP'_I + \beta s_j - y s_I + MK_4. \quad (2\text{-b})$$

$$K_3 + K_4 = 1.$$

$$K_3, K_4 \in \{0,1\}.$$

In the set of *Relation (2)* if:

$K_3 = 0$, demand is low, discount model applies to hotel j,

$K_4 = 1$, since the value of M is a very large number, the constructed relation is considered as a redundant relation whose scale is not considered in the dimensions of the problem. Therefore, taking into account the above assumptions using *Eq. (2-a)*, the demand model is created by considering the price discount. And *Eq. (2-a)* is discarded. In this way, a discount model is applied, which is examined in this article. Therefore, in the relations proposed in the set of *Eqs. (1)* and *(2)*, the discount model is applied only when $K_1 = 0$ and $K_3 = 0$. Considering the demand equations, we can also specify the revenue function for both types of the hotel i and j as follows. If $P'_{i,j} = \gamma_{i,j}\left(P_{i,j} + t\right)$ if γ is applied we can say that $\gamma_{i,j}P'_{i,j}$. So instead of showing $P'_{i,j} = \gamma_{i,j}\left(P_{i,j} + t\right)$ as $\gamma_{i,j}P'_{i,j}$ we show. Now, we write the income model in terms of $K_1 = 0$ and $K_3 = 0$ as follows:

$$\gamma_{i,j} P'_{i,j}; \quad P'_{i,j} = \left(P_{i,j} + t\right). \quad (3)$$

$$R_i = \gamma_i P'_i(\tilde{q}_i) - c_i(\tilde{q}_i) - c'_i - \frac{1}{2}\eta_i s_i^2. \quad (4)$$

$$R_j = \gamma_i P'_j(\tilde{q}_j) - c_j(\tilde{q}_j) - c'_j - \frac{1}{2}\eta_j s_j^2. \quad (5)$$

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Based on *Eqs. (4)* and *(5)*, the revenue model for i or j hotels can be calculated. If for the set of *Eqs. (4)* to *(5)* the value of $\frac{dR_i}{dP'_i}=0$, $\frac{dR_i}{dP'_j}=0$, $\frac{dR_i}{dq_i}=0$ and $\frac{dR_i}{dq_j}=0$ can be calculated then we can calculate the equilibrium points for P'_i, P'_j, q_i and q_j. For this purpose, we first consider $\tilde{q}_{i,j}=q$, $P'_{i,j}=P$ and $\gamma_{i,j}=Y$. In this case, the simplified equation of the income of hotels i and j is presented as *Eq. (6)*.

$$R=\left(Y.P.q\right)-cq-c'-\frac{1}{2}\eta_j s^2. \tag{6}$$

In *Eq. (6)*, the function R is a function with two variables P and q. Now, by deriving the *Eq. (6)* and inserting the estimated demand function, we can calculate the optimal value of each of the considered variables.

$$\frac{dR_{i,j}}{dP'_{i,j}}=Yq=0. \tag{7}$$

According to the value obtained in *Eq. (7)*, and by replacing $Y=\gamma_i$ and $\tilde{q}_i=q$, taking into account $k=0$, then the value of the equilibrium price of the hotel i in the discount conditions will be equal to *Eq. (8)*.

$$P_i'^{*}=(\gamma_i\beta s_i+\gamma_i y s_j-\gamma_i\phi\mu a)/((d-1)y_i^2). \tag{8}$$

Also, the equilibrium price of hotel j at the discount will be equal to *Eq. (9)*.

$$P_j'^{*}=\frac{-a\gamma_j+\gamma_j\mu a-\gamma_j\beta s_j+\gamma_j y s_i}{\left(d-1\right)\gamma_j^2}. \tag{9}$$

Also, by deriving from *Eq. (6)*, the optimal value of demand is calculated as *Eq. (10)*.

$$\frac{dR_{i,j}}{dq_{i,j}}=PY-c=0. \tag{10}$$

According to the value obtained in *Eq. (10)*, by placing $P'^{*}_{(i,j)}$ in this case, the amount of equilibrium demand for hotels i and j in the discount conditions is equal to *Eqs. (11)* and *(12)* will be.

$$q_i^*=\frac{\gamma_i\beta s_i+\gamma_i y s_j-\gamma_i\phi\mu a}{\left(d-1\right)\gamma_i^2}.\gamma_i-c. \tag{11}$$

$$q_j^*=\frac{-a\gamma_j+\gamma_j\mu a-\gamma_j\beta s_j+\gamma_j y s_i}{\left(d-1\right)\gamma_j^2}.\gamma_j-c. \tag{12}$$

4 | Research Results

To show the ability to implement the proposed model, we have considered three-star, four-star and five-star hotels in a tourist area in northern Iran. All these hotels are competing with every star they have to attract tourists. For example, two five-star hotels in the proposed model compete with each other. Some hotels are located in the city center and others close to the beach. Hotels offer a variety of green services and facilities, and some hotels use cost-effective methods that are higher in terms of hotel index, such as hotel i. However, the other hotels probably do not have a specific policy and do not use cost-saving methods such as hotel j. The parameters used in this research have been collected from surveys and interviews with hotel managers and hotel industry experts in Iran. The parameter values for each hotel are summarized in *Table 2*.

Table 2. Preset parameter values of 5-star, 4-star and 3-star hotels.

Paameters	c_i	c_j	c'_i	*	η_i	η_j	f	μ	β	y	d	s_i	s_j	a	γ
5 Star hotel	0.8	0.95	35	45	0.6	0.4	0.75	0.4	7	4	0.3	5	3	400	0.4
4 Star hotel	0.5	0.65	15	20	0.6	0.4	0.75	0.4	7	4	0.2	5	3	290	0.5
3 Star hotel	0.35	0.4	7	10	0.6	0.4	0.75	0.4	7	4	0.2	4	2	180	0.6

Based on the presented *Eqs. (8)*, *(9)*, *(11)* and *(12)*, we calculate the optimal amount of demand and price in terms of discount to calculate the optimal income based on the obtained values. *Table 3* shows the optimal results for the optimal amount of demand, price and income, taking into account the discount conditions.

Table 3. The optimal amount in terms of discount.

		5 Star Hotel	4 Star Hotel	3 Star Hotel
Hotel i	P'^*_i	28.40	10.56	69.01
	q^*_i	37.09	17.99	301.38
	R^*_i	40.38	23.87	555.65
Hotel j	P'^*_j	214	84.80	7141.52
	q^*_j	189.37	94.13	8835.65
	R^*_j	137.40	82.09	6721.11

The study hypothesizes that the government intervenes in possible decisions. In other words, it is assumed that the government as a leader seeks to maximize desirable performance. Therefore, the government can influence the price of hotels by imposing tariffs. For this purpose, the best hotel response based on government tariffs according to the utility relationship considered by Mousavi et al. [26], can be calculated as $U = \Omega.g + (1-\Omega)(\lambda f + (1-\lambda)h$.

In relation to desirability $g = q_i + q_j$; $f = t_i q_i + t_j q_j$ and $h = R_i + R_j$. Also, λ and Ω are two important and effective parameters in the performance of the utility function of the U state. The coefficient λ shows the importance of government revenue over hotel revenue. The coefficient Ω shows the importance of guest demand and attention to the expansion of the tourism industry. *Table 4* shows the effect of λ and Ω on the utility function of government U considering the government tariffs $t_{i,j}$ for 5-star hotels i, j.

According to *Table 4*, by increasing the values of λ and Ω at the same time, the value of the state utility function decreases sharply. Therefore, based on this initial study, it shows that the proposed model is sensitive to the values of λ and Ω. Therefore, their amount can not be reduced quickly .For example, if λ is kept constant and the value of Ω increases, it decreases with less intensity.

In contrast, by holding Ω constant and changing λ, the state utility function changes again with greater intensity. From the results obtained in *Table 3*, it can be concluded that the λ coefficient has a major impact on government profits of hotels. Based on this model, it can be concluded that if the λ coefficient increases, the state utility function decreases, so it is necessary to keep it as close as possible.

Table 4. Parameter changes for 5-star hotels.

	λ	Ω	t_i	t_j	f	U
5 Star Hotel g=95.36;h = 7210.53	0.1	0.1	-9.2	-3.5	-393.952	5814.61
	0.2	0.2	-0.3	9.3	785.472	4759.487
	0.3	0.3	17.3	35.2	3167.648	4226.974
	0.4	0.4	70	113	10321.6	5111.119
	0.5	0.5	4.36	15.51	1361.29	2190.635
	0.6	0.6	-143.4	-198.6	-18355.6	-3194.44
	0.7	0.7	-90	-121	-11211.2	-1638.65
	0.8	0.8	-73.5	-96	-8916.96	-1062
	0.9	0.9	-67	-86	-8000.32	-562.1
	0.1	0.1	-9.2	-3.5	-393.952	5814.61
	0.1	0.2	-9.1	-3.4	-384.416	5179.9
	0.1	0.3	-9	-3.3	-374.88	4545
	0.1	0.4	-8.8	-3.1	-355.808	3910.482
	0.1	0.5	-8.6	-2.9	-336.736	3275.582
	0.1	0.6	-8.3	-2.6	-308.128	2640.682
	0.1	0.7	-7.8	-2.1	-260.448	2005.782
	0.1	0.8	-6.7	-1.8	-223.392	1369.716
	0.1	0.9	-3.6	2	131.584	736.0875
	0.1	0.1	-9.2	-3.5	-393.952	5814.61
	0.2	0.1	-0.5	9.2	774.88	5340.596
	0.3	0.1	17	35	3147.52	5402
	0.4	0.1	69	111.5	10183.84	7569.405
	0.5	0.1	1.04	-1	-73.8176	3221.057
	0.6	0.1	-140	-195	-18014.4	-7122.45
	0.7	0.1	-87.6	-118.5	-10973.9	-4957.15
	0.8	0.1	-70.2	-93	-8627.71	-4904.52
	0.9	0.1	-61.5	-80	-7433.44	-5362.6

In the following, the behavior of the proposed model for five-star, four-star and three-star hotels separately, taking into account the value of $\lambda = 0.01$ and for changes in the value of Ω between 0.1 to 0.9 for each of the five-star hotels, four stars and three stars are examined in *Table 5*.

Table 5. Sensitivity analysis of Ω on five-star, four-star and three-star hotels.

Hotel Type	λ	Ω	t_i	t_j	f	U
5 Star	0.01	0.1	-14	-10.5	-1038.24	6424.774
	0.01	0.2	-13.9	-10.4	-1028.7	5721.582
	0.01	0.3	-13.8	-10.3	-1019.17	5018.371
	0.01	0.4	-13.7	-10.2	-1009.63	4315.141
	0.01	0.5	-13.5	-10.1	-999.04	3611.897
	0.01	0.6	-13.3	-9.8	-971.488	2908.7
	0.01	0.7	-12.8	-9.4	-932.288	2205.483
	0.01	0.8	-12	-8.5	-847.52	1502.278
	0.01	0.9	-9.4	-6.4	-641.984	799.0245
4 Star	0.01	0.1	-10.4	-8	-940.268	8143.856
	0.01	0.2	-10.3	-7.9	-929.054	7251.532
	0.01	0.3	-10.2	-7.85	-922.548	6359.153
	0.01	0.4	-10.1	-7.7	-906.627	5466.817
	0.01	0.5	-9.9	-7.56	-889.849	4574.454
	0.01	0.6	-9.6	-7.3	-859.974	3682.11
	0.01	0.7	-9.2	-6.9	-815.12	2789.75
	0.01	0.8	-8.4	-6	-715.999	1897.41
	0.01	0.9	-13	-3.5	-563.441	1004.925
3 Star	0.01	0.1	-6.4	-5.4	-596.105	6488.839
	0.01	0.2	-6.3	-5.3	-585.508	5779.716
	0.01	0.3	-6.2	-5.2	-574.912	5070.572
	0.01	0.4	-6.1	-5.1	-564.315	4361.406
	0.01	0.5	-5.9	-4.9	-543.121	3652.273
	0.01	0.6	-5.7	-4.6	-513.719	2943.129
	0.01	0.7	-5.3	-4.2	-471.331	2233.966
	0.01	0.8	-4.4	-3.4	-384.169	1524.808
	0.01	0.9	-1.9	-0.8	-111.04	815.661

In the calculations performed, the equilibrium price, equilibrium demand and equilibrium income obtained according to the obtained relations have been used. The results show that in a competitive environment, government tariffs have a significant impact on hotel prices and hotel revenues. We know that the optimal prices for hotels and hotel revenues depend heavily on tariffs. Therefore, the government should carefully consider the hotel's reaction to tariff decisions. In a competitive environment, price changes not only affect the price of the hotel, but also the revenue of the hotel.

5 | Conclusion

The hotel industry is known as the largest subset of the tourism industry. Hotels of different sizes and situations are generally constantly competing with each other to attract tourists. In this regard, unexpected factors have a great impact on the performance of hotels in attracting tourists. For example, the widespread outbreak of the Covid-19 pandemic has affected many businesses around the world, including the tourism industry. In this case, the government, in order to overcome the fears of a decline in the tourism industry due to the pandemic, will impose its discount policies and regulations to develop a competitive advantage in order to improve the performance of the tourism industry. In this article, we have considered an attractive tourist area in the north of Iran, which has a large number of hotels. These hotels compete with each other to attract customers. In the proposed model, there are hotels with the same star that compete with each other with different hotel service indicators. Also, the discount rate on the price of each hotel room varies according to its star. Considering the defined conditions, we considered Stackelberg game with decentralized conditions in the model. In this situation, the leader influences the hotels by offering appropriate tariffs based on their goals and economic situation, and incentive policies in order to improve the hotel index. Therefore, the government helps hotels to attract tourists by setting different tariffs. Because increasing tourists can be a good source of income for hotels. The results show that the discount policies and regulations that the government imposes on hoteliers are able to force them to pay more attention to the competitive issue, which improves the performance of hotels. Because, with the development of the tourism industry and the encouragement of hotels to apply discounts on prices, there will be an increase in demand, so they will earn a significant net income. For future research, the use of dynamic demand models in calculating price and calculating the optimal demand is recommended. Also, the division of domestic and foreign tourists by considering the influential factors related to the elements of the demand equation will be useful.

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Decomposition of Cost Efficiency, Given the Set of Price and Cost Production Possibilities in Data Envelopment Analysis

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Abstract

To evaluate the performance and estimate the efficiency of Decision-Making Units (DMUs) in Data Envelopment Analysis (DEA), the available data are used. These data are usually divided into two categories of inputs and outputs based on their natures. If the price data is also available for inputs, it is necessary to calculate another type of the efficiency called Cost Efficiency (CE). Since the efficiency of units in such a framework is depended on the both quantities of inputs and outputs and also the prices of inputs, it is important to find the sources of cost inefficiency related to each of the factors and plans to address them. In this paper, we intend to present a new decomposition of CE and observed cost versus optimal cost, which are raised from each of the factors involved in the cost inefficiency, in a non-competitive pricing environment which the input price vector for different DMUs can be different. Moreover, for the first time, in parallel with using the PPS based on input and output quantities and introducing some cost inefficiency factors related to this set, we will introduce new sets called price and cost production sets that the first is based on the prices of inputs and output factors, and the second is based on the optimal vectors of inputs and prices obtained from two previous PPS, and then we will introduce other factors of cost inefficiency in the sets. Accordingly, new decomposition for cost inefficiencies will be presented. Also, in the previous analyzes, congestion inefficiency has not been considered as one of the important factors in cost inefficiency. In this study, we also intend to consider the impact of this factor on CE.

Keywords: Data envelopment analysis, Cost efficiency, Non-competitive environment, Cost PPS, Cost decomposition, Congestion.

1 | Introduction

One of the most important concepts in the performance appraisal by Data Envelopment Analysis (DEA) is efficiency. This score indicates the position of a unit relative to an efficient frontier. Usually, based on the acceptance of a series of basic principles, a set called the Production Possibility Set (PPS) is made, and its boundary is called the efficient frontier. This set is based on the information available to the units. This data is usually classified into two categories: inputs and outputs. The first category is usually the resources used to generate the outputs. If, the input prices are also available, the Cost Efficiency (CE) measure will be considered in performance appraisal.

1.1 | Literature Review

In the CE models, the efficiency of the under evaluation unit is compared to a real or virtual unit of the efficient frontier with a lower cost to produce at least the same output as the unit under evaluation.

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Mombini et al. [1] proposed an approach to attain the sustainability radius of the CE considering interval data. Sarab et al. [2] proposed a two-step procedure to maintaining CE in sugar industries under any fluctuation in input costs. To calculate CE in aquaculture using the case of intensive white-leg shrimp farming in Phu Yen province of Vietnam, Long [3] used a two-stage bootstrapping technique. Focusing on CE and running longitudinal case-based research over six years (2014–2019), Piran et al. [4] applied an internal benchmarking analysis for evaluation of economic efficiency of a broiler production system. Lotfi et al. [5] proposed a method to modify the classic CE DEA model in order to investigate the situations of market discounts. Rezaei Hezaveh et al. [6] introduced a cost based PPS in the non-competitive environment to assess Cost, Revenue and Profit Efficiency. Paleckova [7] used two-stage DEA to estimate CE and its determinants of the Czech and Slovak commercial banks within the period of 2005-2015. Soleimani-Chamkhorami [8], used inverse DEA for preserving cost/revenue efficiency of European and American banks. A new decomposition of CE is given in [9] when DMUs are not price takers. A cost minimizing planning problem of a state government in the US were considered by Shiraz et al. [10] in the framework of economic efficiency measures for stochastic data with known input and output prices. Ghiyasi [11] provides the theoretical foundation of the inverse DEA problem when price information is available. Toloo [12] developed a method for finding the most cost efficient DMU when the prices are fixed. Khanjani Shiraz et al. [13] developed a nonparametric methodology for cost-efficiency based on rough set theory to rank and evaluate DMUs when incorporation of data uncertainty. Fang and Li [14] developed models and a base-enumerating algorithm to calculate the upper and lower bounds of CE for each firm in the case of non-unique law of one price while keeping the industry CE optimal. Mozaffari et al. [15] formulated an original DEA-R cost and revenue efficiency models in the case of same price vector for ratio quantities of inputs to outputs. Sahoo et al. [16] states that in a non-competitive market with different input prices, it would be appropriate to use a value-based technology, in which the performance of units can be evaluated in comparison with it. Fang and Li [17] presented a method which can acquire the Pessimistic CE measure in cases with multiple inputs and outputs using the weight restrictions in the form of input cone assurance to determine the lower bound of CE. Camanho and Dyson [18] presented the idea of economic efficiency as a development for Farrell CE in the non-competitive. Jahanshahloo et al. [19] given a method for CE analysis which deals with ordinal data. Camanho and Dyson [20] proposed a process for estimating the bounds of the CE in situations where only a maximal and minimal bounds of input prices can be determined for each DMU. Jahanshahloo et al. [21] suggested a condensed version of [20]'s model with fewer numbers of restrictions and variables. Tone and Tsutsui [22] decomposed observed total cost into the global optimal (minimum) cost and loss due to technical inefficiency in technical PPS, input price difference and inefficient cost mix, which are measured in the cost based PPS.

1.2 | Research Gap and Main Contricutions

It is known that the cost is a function of the amount of inputs and their price. So any inefficiency in the proper use of inputs and inefficiency in the use of appropriate prices leads to cost inefficiency. In the DEA literature, two types of inefficiencies can be attributed to inputs, one is technical inefficiency and the other is congestion. Under normal conditions, an increased input will lead to the increased output, but if one or more input increase as one or more outputs decrease, or else, if one or more inputs decrease as one or more outputs increase, congestion will be said to exist in the inputs. Congestion, to be sure, might not be necessarily the result of a direct association between each input and output since the above mentioned concept of congestion is more comprehensive than the concept of the congestion in economics. In general, however, congestion is said to exist if the increase of an input factor which have cost nature does cause a decrease of outputs which have an income nature. What is considered in cost studies in DEA is often technical inefficiency and the effect of congestion inefficiency on cost performance is overlooked. If price data are present, one can consider the price-related congestion, since the prices have a nature similar to inputs, and if they increase too much, the input may decrease. On the other hand, in the literature on CE, less attention is paid to fixing cost inefficiencies and improving them by fixing price inefficiencies. It is obvious that such an issue makes sense in the presence of different price data between units. If the prices are the same for all units, it is clear that cost inefficiencies can only be attributed to inefficiencies related to resources or inputs.

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Tone [23] and Tone and Tsutsui [22], using the basic principles of DEA, introduced their cost PPS. Their PPS were made by accepting the constant return to scale principle and, in making cost PPS of Tone [23], only the observed prices and outputs were used, and in Tone and Tsutsui [22] modified cost obtained by removing technical inefficiencies in inputs was used in conjunction with outputs, and the modified prices and eliminating other types of technical inefficiencies such as congestion was neglected. Furthermore, if we consider a triple (xj, cj, cjxj) = (observed quantities of inputs, obsereved prices, and observed costs) for CE estimation, then we can also consider a triple (Px, Pc, Pcx) for PPSs. Accordingly, considering the first component of the first triple with the observed outputs leads to the creation of conventional quantity based PPS Px, and also considering the observed costs 'cjxj' along with the observed outputs leads to the creation of the cost PPS of Tone [23]. Therefore, what is expected is to build a PPS using the second component of the first triple, i.e. 'cj' and the observed outputs, and this can be considered our motivation in building the Pc.

From the considerations above, the contribution of this paper in the literature are given in the following:

I. Investigating the effect of congestion related to input quantities and inputs technical inefficiencies on CE by calculating the relevant inefficiencies and excess costs due to these inefficiencies in Px.
II. Construction a new price-based PPS Pc.
III. Investigating the effect of congestion related to prices and price technical inefficiencies on CE by calculating the relevant inefficiencies and excess costs due to these inefficiencies in Pc.
IV. Composing a new set of cost based PPS using optimal inputs and prices, calculating two types of mix and cost allocative efficiency and finally break down CE based on all previous efficiencies, as well as express the observed cost based on the optimal cost and all excess costs incurred by the unit under assessment due to various inefficiencies.

It should be noted that the cost PPS set will be made in this paper is different from Tone [23] and Tone and Tsutsui [22]. As mentioned before, the observed costs were used in Tone [23]. In Tone and Tsutsui [22], the modified costs obtained by the optimal input vectors and the observed prices are used, while in this paper, both the modified inputs and the modified prices will be used to construct the cost PPS. It can be easily proved that the two previous cost PPS are a subset of the cost PPS will be introduced in this paper, and therefore it can be examined that the method proposed in this paper is able to identify more sources of cost inefficiency. In [22] and [23], all inefficiencies in the input and output quantities based PPS are considered as technical. As a result, only the effect of this type of inefficiency on CE and excess cost is mentioned. However, as far as the authors know, for the first time in this paper, the effect of input congestion on cost inefficiency has been considered independently. Also in [22], reducing inputs is a priority to eliminate cost inefficiencies, rather than reducing prices. In this way, first the sources of quantitiy technical inefficiency are eliminated and then the price inefficiency corresponding to the cost point made from the optimal inputs and observed prices in their proposed set of cost based PPS Pcx are eliminated. Methods in [22] and [23] are not able to respond if the reduction of prices and the selection of optimal prices or the investigation of the cost inefficiency and the excess cost imposed due to the non-selection of the appropriate price are desired. It should be noted that depending on the whether fixing input inefficiencies is a priority for the decision maker or fixing price inefficiencies, two decompositions of CE and observed cost will be presented which are considered as a step-by-step path to eliminate inefficiencies. In both presented decomposition, attention has been paid to the elimination of both types of congestion inefficiency and congestion free technical inefficiencies in separate PPSs of quantities and prices.

The paper is organized as follow. The second part is dedicated to some preliminaries. In Section 3, the proposed method for analyzing CE and observed cost is described in stages. In Section 4, by giving two numerical examples and a practical example, we show the applicability of the proposed method. The final section is devoted to conclusions and suggestions for the future studies.

2 | Preliminary

2.1 | Data Envelopment Analysis

A pair of multiple-input $X \in R^m$ and multiple-output $Y \in R^s$ is called an activity, and it is expressed by the notation (X,Y).The set of feasible activities which is called the PPS is given as follow:

$$T = \{(X, Y) \mid X \text{ can produce } Y\}.$$

The following properties is postulated in DEA for PPS T:

I. Inclusion of observations: the observed activity (X_j, Y_j) belongs to T for all $j=1,...,n$.

II. Convexity: if $(X,Y) \in T$ and $(X',Y') \in T$, then $(\lambda X+(1-\lambda)X', \lambda Y+(1-\lambda)Y') \in T$ for all $\lambda \in (0,1)$.

III. Ray unboundedness (constant return to scale): if $(X,Y) \in T$, then $(kX, kY) \in T$ for any positive scalar k.

IV. Free Disposal (monotonicity): if $(X,Y) \in T$ and $X' \geq X \,\&\, Y' \leq Y$, then $(X',Y') \in T$.

V. Minimum extrapolation: if a PPS T' satisfies before postulates, then $T \subset T'$.

The unique empirical PPS T_V has four properties that is defined *Eqs. (1)*, *(2)*, *(4)* and *(5)* as follows[1]:

$$T_V = \left\{(X,Y) \middle| X \geq \sum_{j=1}^{n} \lambda_j X_j \,, Y \leq \sum_{j=1}^{n} \lambda_j Y_j \,, \sum_{j=1}^{n} \lambda_j = 1 \,, \lambda_j \geq 0 \,; j=1,...,n \right\}.$$

Sometimes this set is called the PPS of the BCC model, or the PPS with variable returns to scale technology [24].

The boundary of the PPS is called "efficient frontier". Each DMU on the efficient frontier is called efficient, and the others are inefficient. There are various strategies that can determine the efficiency position of an under evaluation. One of the most important of them aims to minimize inputs while satisfying at least the given output levels. This is called the input-orientation. There is another type called the output-orientation that attempts to maximize outputs without requiring more of any of the observed input values. To evaluate the relative efficiency of DMU_o in T_v in the input orientation, the following BCC model can be solved:

$$\begin{aligned}
&\text{Min} \quad \theta - \varepsilon(1^T S^- + 1^T S^+),\\
&\text{s.t.}\\
&\quad \sum_{j=1}^{n} \lambda_j X_j + S^- = \theta X_o,\\
&\quad \sum_{j=1}^{n} \lambda_j Y_j - S^+ = Y_o,\\
&\quad \sum_{j=1}^{n} \lambda_j = 1,\\
&\quad \lambda_j \geq 0, \quad j=1,...,n,\\
&\quad S^- \geq 0, S^+ \geq 0.
\end{aligned} \tag{1}$$

[1] It is necessary to explain that by accepting all or some of the principles, and as well as changes in some principles such as ray unboundedness, different PPSs that led to models BCC-CCR, CCR-BCC, FDH, CHD [24] and etc. can be made, which the discussion of which is beyond the scope of this paper.

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Where ε is a non-Archimedean value, meaning that it is smaller than any small positive number. For more information about Epsilon [25].

The output orientation version of BCC is:

$$
\begin{aligned}
&\text{Max} \quad \varphi+\varepsilon(1^T S^- + 1^T S^+),\\
&\text{s.t.}\\
&\quad \sum_{j=1}^{n} \lambda_j X_j + S^- = X_o,\\
&\quad \sum_{j=1}^{n} \lambda_j Y_j - S^+ = \varphi Y_o,\\
&\quad \sum_{j=1}^{n} \lambda_j = 1,\\
&\quad \lambda_j \geq 0, \quad j=1,\dots,n,\\
&\quad S^- \geq 0, \quad S^+ \geq 0.
\end{aligned}
\tag{2}
$$

2.2 | Congestion

Noura et al. [26] presented a relatively simple approach for finding congestion. Using this method, the input congestion of DMU_o will be diagnosed as follows:

The set E of efficient units of *Model (2)*, will be considered as $E=\{j|\varphi_j^*=1\}$. Among the efficient units of the E set, the units with the highest values in at least one input will be selected and the input related to that unit will be specified with x_i^{max}. There is congestion in DMU_o if and only if there is at least one of the following conditions in the optimal solution of *Model (2)*, $(\theta^*, \lambda^*, s^{-*}, s^{+*})$:

I. $\varphi_o^* > 1$ and at least in one component, $x_{io} > x_i^{max}$.

II. At least in one component, $s_r^{+*} > 0$ and at least in one component, $x_{io} > x_i^{max}$.

In this case, the congestion value will be $S_i^c = x_{io} - x_i^{max}$ in the i^{th} input.

3 | Methodology

This section consists of two subsections. In the first part, we will explain the steps of the proposed method. In order to maintain the integrity of the proposed algorithm structure, we present in the second section some discussions about the PPSs, the models used, as well as some theoretical material that scientifically supports the proposed method.

3.1 | Steps of the Proposed Method

Step 1. Calculating the observed cost for DMUo with the input x_o and the observed price C_o:

$$C_o = \sum_{i=1}^{m} c_{io} x_{io}.$$

Step 2. Constructing the PPSs P_x and P_c:

$$P_x = \{(X,Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j=1,\dots,n\}.$$

$$P_c=\{(C,Y)\mid C\geq\sum_{j=1}^{n}\lambda_j C_j, Y\leq\sum_{j=1}^{n}\lambda_j Y_j,\sum_{j=1}^{n}\lambda_j=1,\lambda_j\geq 0, j=1,\ldots,n\}.$$

Step 3. Calculations in P_x and P_c :

Part 1: Calculations in P_x :

I. Calculating the input congestion S_i^c of the i^{th} input for (x_o, y_o) using *Model (2).*

II. Calculating the cost of the congestion-free input $x_{io}{}^c = x_{io} - s_{io}^c \ (i=1,\ldots, m)$, with the observed price, c_o^1 as follows:

$$C_o{}^1=\sum_{i=1}^{m}c_{io}x_{io}^c.$$

III. Calculating the excess cost caused by congestion in the inputs:

$$L_o^{\text{Input congestion}}=C_o-C_o{}^1.$$

IV. Calculating the input congestion efficiency:

$$\text{Input congestion efficiency}=\frac{C_o{}^1}{C_o}.$$

V. Finding the input technical efficiency of the unit (x_o^c, y_o) with the components of congestion-free x_o^c vector, using *Model (1).*

VI. Obtaining the technical efficient point (input projection point) $x_o^* = \theta_x^* x_o^c - s_x^{-*}$ using the optimal solution $(\theta_x^*, \lambda_x^*, s_x^{-*}, s_x^{+*})$ of *Model (1).*

VII. Calculating the cost of the technical efficient point x_o^*, with the observed price c_o as:

$$C_o{}^2=\sum_{i=1}^{m}c_{io}x_{io}^*.$$

VIII. Calculating the excess cost caused by technical inefficiency in the inputs:

$$L_o^{\text{Tech. input}}=C_o{}^1-C_o{}^2.$$

IX. Definition of the (free congestion) input technical efficiency as:

$$\text{Input technical efficiency}=\frac{C_o{}^2}{C_o{}^1}.$$

Part 2: Calculations in P_c :

I. Calculating the input congestion S_i^c of the i^{th} price for (c_o, y_o) using *Model (2).*

II. Calculating the cost of the congestion-free price $c_{io}{}^c = c_{io} - s_{io}^c$ (i = 1,…, m), with the observed input, x_o as follows:

$$C_o{}^3=\sum_{i=1}^{m}c_{io}^c x_{io}.$$

III. Calculating the excess cost caused by congestion in the prices:

$$L_o^{\text{Price congestion}} = C_o - C_o^{\ 3}.$$

IV. Calculating the price congestion efficiency:

$$\text{Price congestion efficiency} = \frac{C_o^{\ 3}}{C_o}.$$

V. Finding the price technical efficiency of the unit $\left(c_o^c, y_o\right)$ with the components of congestion-free c_o^c vector, using *Model (1)*.

VI. Obtaining the price efficient point (price projection point) $c_o^* = \theta_c^* c_o^c - s_c^{-*}$ using the optimal solution $\left(\theta_c^*, \lambda_c^*, s_c^{-*}, s_c^{+*}\right)$ of *Model (1)*.

VII. Calculating the cost of the price efficient point c_o^*, with the observed input x_o as:

$$C_o^4 = \sum_{i=1}^{m} c_{io}^* x_{io}.$$

VIII. Calculating the excess cost caused by price inefficiency in the inputs:

$$\text{Price technical efficiency} = \frac{C_o^{\ 4}}{C_o^{\ 3}}.$$

IX. Definition of the (free congestion) price technical efficiency as:

$$L_o^{\text{Tech. Price}} = C_o^3 - C_o^4.$$

Step 4. Considering the various combinations of x_i^c, x_i^*, c_i^c and c_i^*:

Part 1: Calculating the costs.

I. The cost value of congestion-free input x_i^c with the technical efficient price c_i^* will be as:

$$C_o^{\ 5} = \sum_{i=1}^{m} c_{io}^* x_{io}^c.$$

II. The cost value of technical efficient input x_i^* with the congestion-free price c_i^c will be as follows:

$$C_o^{\ 6} = \sum_{i=1}^{m} c_{io}^c x_{io}^*.$$

III. The cost value of technical efficient input x_i^* with the technical efficient price c_i^* will be as follows:

$$C_o^{\ 7} = \sum_{i=1}^{m} c_{io}^* x_{i0}^*.$$

Part 2: Calculating the excesses.

I. Calculating the excess cost caused by price congestion when the inputs are free of inefficiency (optimal inputs):

$$L_o^{\overline{\text{Price congestion}}} = C_o^{\ 2} - C_o^{\ 6}.$$

II. Calculating the excess cost caused by price inefficiency when the inputs are free of inefficiency (optimal inputs):

$\overline{L_o^{\text{Tech price}}} = C_o^{\,6} - C_o^{\,7}$.

III. Calculating the excess cost caused by input congestion when the prices are free of inefficiency (optimal prices):

$\overline{L_o^{\text{Input congestion}}} = C_o^{\,4} - C_o^{\,5}$.

IV. Calculating the excess cost caused by input inefficiency when the prices are free of inefficiency (optimal prices):

$\overline{L_o^{\text{Tech. input}}} = C_o^{\,5} - C_o^{\,7}$.

The sign is to emphasize that the inefficiencies of one of the factors have been eliminated or the optimal value of one of the factors has been considered.

Part 3: Defining the efficiencies.

I. Definition of the price congestion efficiency when the inputs are free of inefficiency (optimal inputs):

$$\overline{\text{Price congestion efficiency}} = \frac{C_o^{\,6}}{C_o^{\,2}}.$$

II. Definition of the price technical efficiency when the inputs are free of inefficiency (optimal inputs):

$$\overline{\text{Price technical efficiency}} = \frac{C_o^{\,7}}{C_o^{\,6}}.$$

III. Definition of the input congestion efficiency when the prices are free of inefficiency (optimal Prices):

$$\overline{\text{Input congestion efficiency}} = \frac{C_o^{\,5}}{C_o^{\,4}}.$$

IV. Definition of the input technical efficiency when the prices are free of inefficiency (optimal prices):

$$\overline{\text{Price congestion efficiency}} = \frac{C_o^{\,6}}{C_o^{\,2}}.$$

Note that one can consider the combination of x_i^c and c_i^c, which has been neglected in this paper.

Step 5. Constructing the PPS P_{cx} :

After removing the congestion and the price technical inefficiencies in P_x and P_c, and finding the optimal inputs x^* and prices c^* for DMUs, the new PPS P_{cx} which is a cost based technology is produced as follows:

$$P_{cx} = \{(CX, Y) \mid CX \geq \sum_{j=1}^{n} \lambda_j x_j^* c_j^*, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n\}.$$

Step 6. Calculations in P_{cx} :

I. Obtaining the input of projection point $x_o = \theta_{cx}{}^* c_o^* x_o^* - s_c x^{-*}$ using the optimal solution $(\theta_{cx}^*, \lambda_{cx}^*, s_{cx}^{-*}, s_{cx}^{+*})$ from *Model (1)* for $(c_o^* x_o^*, y_o)$.

II. Calculating the cost for the technical efficient input $\hat{x}_o$:

$$C_o^8 = \sum_{i=1}^{m} \hat{x}_{io}.$$

III. Defining the Mix CE as:

$$\text{Mix Cost efficieny}=\frac{C_o^{\,8}}{C_o^{\,7}}.$$

IV. Calculating the excess cost caused by mix cost inefficiency:

$$L_o^{\text{Mix}}=C_o^{\,7}-C_o^{\,8}.$$

V. Finding the unit with the least cost C_o^9 in P_{cx} using the following *Model (3)*:

$$
\begin{aligned}
& C_o^{\,9}=\min e\hat{x}, \\
& s.t. \\
& \quad \sum_{j=1}^{n}\mu_j x_{ij}+t_i^-=x_i, \quad i=1,\dots,m, \\
& \quad \sum_{j=1}^{n}\mu_j y_{rj}-t_r^+=y_r, \quad r=1,\dots,s, \\
& \quad \sum_{j=1}^{n}\mu_j=1, \\
& \quad t_i^-\geq 0, \quad i=1,\dots,m, \\
& \quad y_r, t_r^+\geq 0, \quad r=i,\dots,s\ , \\
& \quad \mu_j\geq 0, \quad j=1,\dots,n.
\end{aligned}
\tag{3}
$$

VI. Defining the allocative efficiency as:

$$\text{Allocative efficiency}=\frac{C_o^{\,9}}{C_o^{\,8}}.$$

VII. Defining the excesses of cost caused by allocative inefficiency:

$$L_o^{\text{Allocative}}=C_o^{\,8}-C_o^{\,9}.$$

Step 7. Decompositions of cost and CE:

Strategy 1. Preferring the input reduction on price reduction.

$$C_o=L_o^{\text{input congestion}}+L_o^{\text{tech. input}}+L_o^{\overline{\text{price congestion}}}+L_o^{\overline{\text{tech. price}}}+L_o^{\text{Mix}}+L_o^{\text{Allocative}}+C_o^{\,9}.$$

$$
\begin{aligned}
\text{Cost eff.}=&\text{Input congestion eff.}\times\text{Input tech. eff}\times\overline{\text{Price congestion eff.}} \\
&\times\overline{\text{Price tech. eff.}}\times\text{Mix ineff.}\times\text{Allocative eff.}
\end{aligned}
$$

Strategy 2. Preferring the price reduction on input reduction.

$$C_o=L_o^{\text{price congestion}}+L_o^{\text{tech. price}}+L_o^{\overline{\text{input congestion}}}+L_o^{\overline{\text{tech. input}}}+L_o^{\text{Mix}}+L_o^{\text{Allocative}}+C_o^{\,9}.$$

$$
\begin{aligned}
\text{Cost eff.}=&\text{Price congestion eff.}\times\text{Price tech. eff.}\times\overline{\text{Input congestion eff.}} \\
&\times\overline{\text{Input tech. eff.}}\times\text{Mix eff.}\times\text{Allocative eff.}
\end{aligned}
$$

As it can be seen, the CE is dependent to different factors such as the input and price congestion, the input and price efficiency, the mix and the allocative efficiency. *Fig. 1* shows the steps of the proposed cost and CE decomposition method.

Step 1. Calculating the observed cost for DMUo with the input and the observed price.

Step 2. Constructing the (PPS) P_x and P_c.

Step 3. Calculations in P_x and P_c.

- Finding the congestion free and technical efficient points and their costs in P_x and P_c.
- Calculating the input (price) congestion and technical efficiencies in P_x and P_c.
- Finding the excess costs due to congestion and technical in efficiencies.

Step 4. Considering the various combinations of x_i^c, x_i^*, c_i^c and c_i^*.

- Calculating the costs, excesses and efficiencies of the cost points.

Step 5. Constructing the PPS P_{cx} using optimal inputs and prices.

Step 6. Calcaulations in P_{cx}.

- Finding the projection points and optimal cost point.
- Calculating the mix and allocative efficiencies.
- Calculating the exess costs due to mix and allocative inefficiencies.

Step 7. Decompositions of cost and cost efficiency.

Fig. 1. Overview of the proposed method.

3.2 | Theoretical Discussion

During the steps of the proposed algorithm, 3 PPS were used, the first set (P_x) being the same PPS with variable return to scale Tv. As stated in the background, this set is made by accepting a series of principles and taking into account the observed input and output quantities of the units. As mentioned before, Tone [23] and Tone and Tsutsui [22], using the basic principles of DEA, introduced their cost PPS. Their PPS were made by accepting the constant return to scale. In contrast, this paper is cautious about accepting the principle of constant return to scale in the presence of prices and therefore costs, and PPSs are based on variable return to scale.

Using the same principles for observed prices and outputs, the Pc set is created. During the proposed algorithm, in Part 1 of Step 3 and the first sentence of Part 2 of Step 3, to investigate the presence of congestion in P_x and Pc, the method of Noura et al. [26] is used, which it uses *Model (2)* itself. Clearly *Model (2)* is always feasible. But the question is whether *Model (1)* is feasible in evaluating virtual units with free of congestion inputs and prices in the fifth sentence of Part 1 of Step 3 and the fifth sentence of Part 2 of Step 3 of the proposed method?

Theorem 1. *Model (1)* is always feasible in evaluating units (x_o^c, y_o) and $\left(c_o^c, y_o\right)$.

Proof: we consider the feasibility of *Model (1)* for the evaluation of (x_o^c, y_o). the proof is the same for $\left(c_o^c, y_o\right)$.

Suppose $(\theta_o^*, \lambda_o^*, s_o^{-*}, s_o^{+*})$ is the optimal solution of *Model (1)* in the evaluation $\left(x_o, y_o\right)$. Then

$$\sum_j \lambda_{oj}^* x_j + s_o^{-*} = \theta_o^* x_o\,,\ \sum_j \lambda_{oj}^* y_j - s_o^{+*} = y_o\,,\ \sum_j \lambda_{oj}^* = 1\,, \lambda_{oj}^* \geq 0\,.$$

On the other hand as $x_o{}^c = x_o - s_o^c$, then $x_o = x_o{}^c + s_o^c$, and therefore $\sum_j \lambda_{oj}^* x_j + s_o^{-*} = \theta_o^* x_o = \theta_o^*(x_o{}^c + s_o^c)$.

As a result $\sum_j \lambda_{oj}^* x_j + s_o^{-*} - \theta_o^* s_o^c = \theta_o^* x_o{}^c$. It is trivial that $s_o^{-*} - \theta_o^* s_o^c \geq 0$. Since on contrary supposing $s_o^{-*} - \theta_o^* s_o^c < 0$ results $s_o^{-*} < \theta_o^* s_o^c$ which is in contrast with max slack of s_o^{-*} in evaluating $\left(x_o, y_o\right)$ using *Model (1)*. So $(\theta_o^*, \lambda_o^*, s_o^{-*} - \theta_o^* s_o^c, s_o^{+*})$ is a feasible solution for the evaluation of (x_o^c, y_o) using *Model (1)*.

After obtaining points with no congested inputs in P_x and prices with no congested prices in P_c, projection points are obtained for such points in the sixth sentence of Part 1 of Step 3 and the sixth sentence of Part 2 of Step 3. To obtain projections, the optimal solution of *Model (1)* in evaluating these points has been used. It can be easily proved that the optimal value is always positive and therefore *Model (1)* will have a finite optimal value in evaluating these points. Therefore, such points are always accessible. As a result, input and price technical efficiencies can be calculated based on them. Also in 1-6, the optimal solution obtained from *Model (1)* in the cost unit evaluation $(c_o^* x_o^*, y_o)$ in Pcx is used to obtain a projection point, which can be concluded with a similar argument that these points are also available. Therefore, mix efficiencies can be calculated based on them. On the other hand, in 6-5 we used *Model (3)* to find the lowest cost point in Pcx. Since $\hat{x}_i = \hat{x}_{io}$, $y_r = y_{ro}$, $\mu_o = 1$, $\mu_j = 0\left(j \neq o\right)$, $t_i^- = 0 (for\ all\ i)$ and $t_r^+ = 0 (for\ all\ r)$ apply in the model, so this model is also always feasible.

In all steps of the algorithm, different efficiencies are introduced based on the defined cost values. It can be easily shown that these values are in the range (0, 1] and also the excess costs defined in the algorithm are all non-negative.

Theorem 2. The following efficiencies are all in the range (0, 1].

- *Input congestion efficiency.*
- *Input technical efficiency.*
- *Price congestion efficiency.*
- *Price technical efficiency.*
- *Mix efficiency.*
- *Allocative efficiency.*

Proof: We express the proof for input congestion efficiency. It is the same for others and we refrain from expressing them. By definition Input congestion efficiency is C_o^1/C_o. On the other hand $C_o^1 = \sum_{i=1}^m c_{io} x_{io}^c = \sum_{i=1}^m c_{io}\left(x_{io} - s_{io}^c\right) = \sum_{i=1}^m c_{io} x_{io} - \sum_{i=1}^m c_{io} s_{io}^c$. As $c_{io} \geq 0\ , for\ all\ i$ and $s_{io}^c \geq 0\ , for\ all\ i$, So, $C_o^1 = \sum_{i=1}^m c_{io} x_{io} - K = C_o - K$. Then $C_o^1 \leq C_o \Rightarrow \dfrac{C_o^1}{C_o} \leq 1$. On the other hand, always $C_o^1 > 0$ (obviously), therefore $C_o^1 / C_o > 0$.This completes the proof. So input congestion efficiency in the range (0, 1].

Theorem 3. The following excess cost values are all non-negative.

- *Excess cost caused by congestion in the inputs (* $L^{Input\ congestion}$ *).*
- *Excess cost caused by technical inefficiency in the inputs (* $L^{Tech.\ input}$ *).*
- *Excess cost caused by congestion in the prices (* $L^{Price\ congestion}$ *).*
- *Excess cost caused by technical inefficiency in the prices (* $L^{Tech.\ price}$ *).*
- *Excess cost caused by cost mix inefficiency (* L^{Mix} *).*
- *Excess cost caused by cost allocative inefficiency (* $L^{Allocative}$ *).*

Proof: we show that $L^{Input\ congestion} \geq 0$. The argument is similar for the rest. By definition $L^{Input\ Congestion} = C_o - C_o^1$. Given that in the process of proving *Theorem 2* it was shown that $C_o^1 \leq C_o$, then $C_o - C_o^1 \geq 0$, thus $L^{Input\ congestion} \geq 0$.

According to the above theorems, it can be concluded that all efficiencies, excess values and projection points are all calculable and well-defined, and therefore CE and observed cost can be decomposed based on these values.

4 | Examples

Example 1. *Table 1* shows the set of 6 DMU, which have one input and one output, and the price of the input unit has been specified.

Table 1. Data of 6 DMUs.

Dmu	Input x_j	Output y_j	Price of Input c_j	Observed Cost x_jc_j
A	2	1	4	8
B	3	3	1.5	4.5
C	4	4	2.5	10
D	6	4	3	18
E	7	3	3	21
F	5	2	5	25

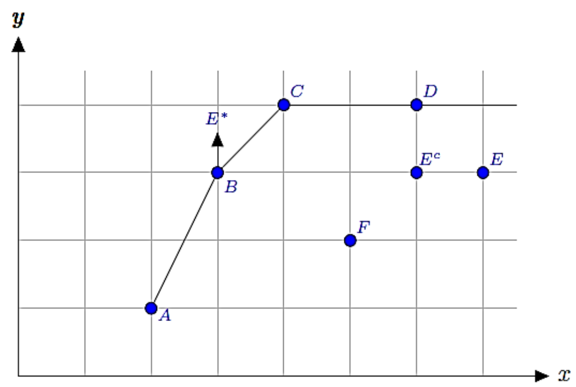

Fig. 2. PPS made by inputs and outputs of DMUs (P_x).

Fig. 2 shows the PPS P_x resulting from the inputs and outputs quantities for these 6 DMUs assuming variable return to scale technology. Clearly, unit's E and F in this set are technically inefficient, and the other units are technically efficient. Consider unit E. One can see that unit E has a congestion in its input. By decreasing one unit of congestion from the input of unit E, the E^c point with value 6 is obtained (See *Fig. 2*). After removing the technical inefficiency of the congestion free point E^c, the can reach to the frontier point E^*.

In the second column of *Table 2*, the congestion free input value for 6 DMUs can be seen. In the third column, there is cost of these inputs. In the two last columns of the *Table 2*, the efficient technical inputs and their costs can be seen which are obtained by removing input technical inefficiencies. The information in this table is derived from some Part 2 calculations of the third step of the proposed algorithm. The values related to the excesses costs and efficiencies of each of the points without congestion and technical inefficiencies in P_x will be presented in *Table 4* to *Table 7*.

Table 2. The congestion free and technical efficient inputs and their costs in P_x.

DMU	x_j^c	$x_j^c c_j$	x_j^*	$x_j^* c_j$
A	2	6	2	6
B	3	4.5	3	4.5
C	4	10	4	10
D	6	18	4	12
E	6	18	3	9
F	5	25	2.5	12.5

It can be noted that in the calculation of the costs in *Table 2*, the observed prices are used. So it may there is some inefficiencies in the prices. On the other hand, considering the prices and outputs, one can form the P_c (*Fig. 3*).

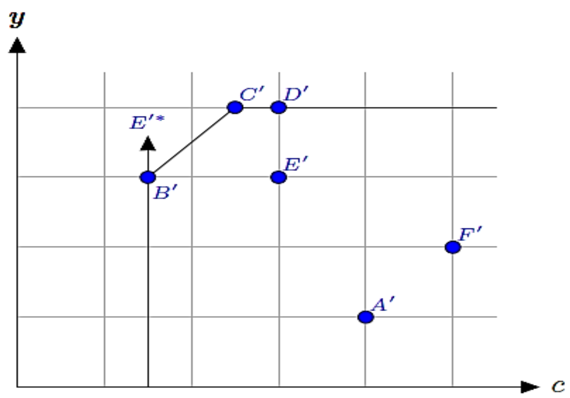

Fig. 3. PPS made by prices and outputs of DMUs (P_c).

Table 3, the first column, shows the prices after congestion removed (A'^c and F'^cin *Fig. 3*).

This time, we consider the process in Part 2 of the third step to obtain congestion and technical inefficiency free prices. It is clear that units A' and F' have a congestion of 2 and 1 units, respectively, while other units E' and D' have only technical inefficiencies in the first input price. Therefore, congestion free points corresponding to price based units A', F', E'and D' will have value 3 in their input. The values are given in column 2 of *Table 3*. In the third column of this table, the corresponding prices for the points for which price congestion inefficiency has been eliminated are obtained using the observed input vector corresponding to each unit. Columns 4 and 5 of *Table 3* show the optimal prices and corresponding costs. As can be seen from *Fig. 3*, there is no price inefficiency for units B'and C', as they are on the Pc efficiency frontier. Therefore, their input price vector is in the optimal position. However, the values of the second and fourth columns for other units have changed, indicating the existence of costly technical inefficiencies after the elimination of congestion inefficiencies (if any). In the last three columns of *Table 3*, the cost values for the various combinations of modified inputs and outputs, which correspond to the first part of Step 4 of the proposed algorithm, are given.

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Table 3. The congestion free prices, technical efficient prices and their corresponding costs in P_c.

DMU	c_j^c	$x_jc_j^c$	c_j^*	$x_jc_j^*$	$x_j^*c_j^c$	$x_j^cc_j^*$	$x_j^*c_j^*$
A'	3	6	1.5	3	6	3	3
B'	1.5	4.5	1.5	4.5	4.5	4.5	4.5
C'	2.5	10	2.5	10	10	10	10
D'	3	18	2.5	15	12	15	10
E'	3	21	1.5	10.5	9	9	4.5
F'	3	15	1.5	7.5	7.5	7.5	3.75

The values in the last column of *Table 3*, obtained using the inputs and optimal prices of Step 1 to Step 3, are the basis for constructing the P_{cx} cost PPS. The corresponding units for each of the A-F units in P_{cx} are indicated by the $^{-}$ symbol in *Fig. 4*. As can be seen in the figure, and of course it was expected in advance, since most of the inefficiencies of the units in the two sets P_x and P_c have been eliminated, all the units in this space are on the efficient frontier and therefore do not have Mix inefficiencies. In this set, unit $\bar{A}$ is clearly a frontier unit with the lowest cost, and therefore it can be introduced as a cost-efficient point, and the allocative efficiencies of other units $\bar{B}-\bar{F}$ can be found according to that unit. The operations related to investigating the existence of mix inefficiencies, finding the CE point and calculating the allocative efficiencies are performed based on Step 5 of the proposed algorithm.

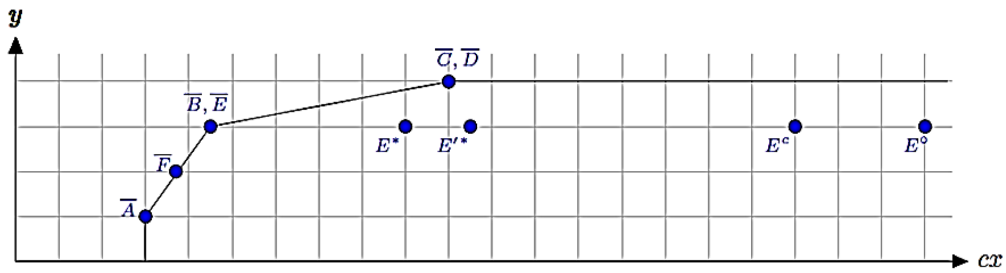

Fig. 4. PPS P_{cx}.

In this case, all cost inefficiencies of other units can be calculated based on a comparison with this cost efficient unit. Also, the amount of excess costs of all units can be analyzed in comparison with this point. *Table 4* shows the excess costs imposed due to various inefficiencies, assuming that input correction takes precedence over price correction. *Table 5* presents the excess costs imposed due to various inefficiencies, assuming that price correction takes precedence over input correction.

Table 4. Excess costs when input reduction is preferred.

DMU	$L_o^{\text{Input congestion}}$	$L_o^{\text{Tech.input}}$	$L_o^{\text{Price congestion}}$	$L_o^{\text{Tech price}}$	L_o^{Mixed}	$L_o^{\text{Allocatice}}$
A	2	0	0	3	0	0
B	0	0	0	0	0	1.5
C	0	0	0	0	0	7
D	0	6	0	2	0	7
E	3	9	0	4.5	0	1.5
F	0	12.5	5	3.75	0	0.75

Table 5. Excess costs when price reduction is preferred.

DMU	$L_o^{\text{Price congestion}}$	$L_o^{\text{Tech. price}}$	$L_o^{\text{Input congestion}}$	$L_o^{\text{Tech.input}}$	L_o^{Mixed}	$L_o^{\text{Allocatice}}$
A	2	3	0	0	0	0
B	0	0	0	0	0	1.5
C	0	0	0	0	0	7
D	0	3	0	5	0	7
E	0	10.5	1.5	4.5	0	1.5
F	10	7.5	0	3.75	0	0.75

Table 6, in turn, shows the decompositions of inefficiencies when the priority is the removal of the input inefficiencies, and *Table 7* shows those when one tries to remove the price inefficiencies as the first priority.

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Table 6. Efficiency decomposition when input reduction is preferred.

Dmu	Input Congestion EFF.	Input Tech. EFF.	Price Congestion EFF.	Price Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	0.75	1	1	0.50	1	1	0.38
B	1	1	1	1	1	0.67	0.67
C	1	1	1	1	1	0.30	0.30
D	1	0.67	1	0.83	1	0.30	0.17
E	0.86	0.50	1	0.50	1	0.67	0.14
F	1	0.50	0.60	0.50	1	0.80	0.12

Table 7. Efficiency decomposition when price reduction is preferred.

Dmu	Price Congestion EFF.	Price Tech. EFF.	Input Congestion EFF.	Input Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	0.75	0.50	1	1	1	1	0.38
B	1	1	1	1	1	0.67	0.67
C	1	1	1	1	1	0.30	0.30
D	1	0.83	1	0.67	1	0.30	0.17
E	1	0.50	0.86	0.43	1	0.67	0.14
F	0.60	0.50	1	0.50	1	0.80	0.12

Fig. 3 shows the cost decomposition of unit E. If the priority is the removal of input efficiencies, the E^o point will be the cost of observed input with observed price, E^c will be the cost of congestion-free with observed price, E^* will be the cost of technical efficient input with the observed price, and $\bar{E}$ will be the cost of technical efficient input with the technical efficient price. In this condition, the decomposition of costs will be as follows:

$$C_{\mathrm{E}} = 21 = 0 + 10.5 + 1.5 + 4.5 + 0 + 1.5 + 3.$$

Moreover, if the priority is to remove the price efficiency, the E^o point will be the cost of observed input with the observed price and also the cost of observed input with the congestion-free price (since there is no congestion in the E price), $E^{*\prime}$ will be the cost of observed input with the technical efficient price, E^* will be the cost of congestion-free input with the technical efficient price, and $\bar{E}$ will be the cost of technical efficient input with the technical efficient price. In this case, the decomposition of costs will be as follows:

$$C_{\mathrm{E}} = 21 = 3 + 9 + 0 + 4.5 + 0 + 1.5 + 3.$$

In the above example, the points obtained the multiples of technical projections of inputs and prices in the P_{cx} space are efficient in this space, and as it was said, this is because all the input and price efficiencies have been already eliminated in the P_x and P_c spaces. In some cases, these points may not be efficient in the P_{cx} space and the remaining inefficiency might not be caused by inefficient inputs or prices, and this type of inefficiency referred to as mix efficiency in the present study. In the following example, this type of inefficiency can be seen.

Example 2. *Table 8* shows the set of six DMU which has two inputs and one output, 1 for all the units, along with the input prices. *Table 9* shows the cost decomposition when the priority is to decrease the input. However, *Table 10* shows the cost decomposition when the priority is to decrease the price. As it can be seen, in this example, after the technical and congestion inefficiencies are removed in the input space and price space, some inefficiencies will continue to exist, and *Tables 9* and *10* show the cost shortages caused by these Mix inefficiencies in the sixth column.

Table 8. Data of DMUs (Example 2).

DMU	Input 1	Input 2	Output	Price of Input 1	Price of Input 2
A	3	1	1	2	3
B	2	2	1	4	2
C	4	3	1	0.5	2
D	5	1	1	1	3
E	3	4	1	1.5	1
F	6	2	1	2	4

Table 9. Cost decomposition of DMUs in Example 2, when input reduction is preferred.

DMU	$L_o^{input\ congestion}$	$L_o^{tech.input}$	$L_o^{price\ congestion}$	$L_o^{tech\ price}$	L_o^{Mix}	$L_o^{allocatice}$
A	0	0	0	4.5	0	0.73
B	0	0	0	7	0.83	0.4
C	0	3.43	0	0	0	0.8
D	0	2	0	2.23	0	0
E	0	3.5	0	0	0.83	0.4
F	0	10	0	5.84	0	0.93

Table 10. Cost decomposition of DMUs in Example 2, when price reduction is preferred.

DMU	$L_o^{price\ congestion}$	$L_o^{tech.price}$	$L_o^{input\ congestion}$	$L_o^{tech.input}$	L_o^{Mixed}	$L_o^{allocative}$
A	0	4.5	0	0	0	0.73
B	0	7	0	0	0.83	0.4
C	0	0	0	3.43	0	0.8
D	0	2.97	0	1.26	0	0
E	0	0	0	0.5	0.83	0.4
F	0	11.68	0	4.16	0	0.39

Example 3. In order to show the practicability of this decomposition, we use the example presented by Tone and Tsutsui [22]. *Table 11* shows the data of 12 hospitals, consisting of two inputs (physicians and nurses) and the price of each input unit (wages of each physician and nurse) and two outputs (number of hospitalized patient and outpatient). *Table 12* and 14 shows the decomposition of cost and efficiency for the case in which the priority is to remove the input inefficiencies or decrease the input, i.e. to decrease the number of physicians and nurses, than to decrease their wages. *Tables 13* and *15* shows the decomposition of cost and efficiency for the case in which the priority is to remove the price inefficiencies, i.e., to decrease the wages of physicians and nurses, than to decrease the workforce consisting of physicians and nurses. Unit E, with the best performance among all the units, has only the technical inefficiency, i.e., Hospital E just has to decrease the workforce in order to achieve efficiency.

Table 11. Data of 12 hospitals.

	Inputs					Outputs	
DMU	Number of Inpatient	Number of Outpatient	PER Nurse's FEE	Number of Nurses	Per Doctor's FEE	Number of Doctors	Number of Inpatient
A	90	100	100	151	500	20	90
B	50	150	80	131	350	19	50
C	55	160	90	160	450	25	55
D	72	180	120	168	600	27	72
E	66	94	70	158	300	22	66
F	90	230	80	255	450	55	90
G	88	220	100	235	500	33	88
H	80	152	85	206	450	31	80
I	100	190	76	244	380	30	100
J	100	250	75	268	410	50	100
K	147	260	80	306	440	53	147
L	120	250	70	284	400	38	120

Table 12. Cost decomposition of DMUs in real example, when input reduction is preferred.

DMU	$L_o^{input\ congestion}$	$L_o^{tech.input}$	$L_o^{price\ congestion}$	$L_o^{tech\ price}$	L_o^{Mixed}	$L_o^{allocatice}$
A	0	0	4220	2610	0	2720
B	0	0	0	1577.9	0	2.1
C	0	2998.8	1650	2625.15	0	2826.05
D	0	0	11040	3165	0	6605
E	0	2110	0	0	0	0
F	900	5552.1	434.3	4688.27	0	18025.33
G	0	0	6680	4165	0	13605
H	0	6330.4	1070.4	3007.4	0	5501.8
I	0	1906.84	0	1765.53	0	10721.63
J	0	0	0	1840	0	23210
K	0	0	0	0	0	32250
L	0	0	0	0	0	19530

Table 13. Cost decomposition of DMUs in real example, when price reduction is preferred.

DMU	$L_o^{price\ congestion}$	$L_o^{tech.price}$	$L_o^{input\ congestion}$	$L_o^{tech.input}$	L_o^{Mixed}	$L_o^{allocative}$	$L_o^{price\ congestion}$
A	23012	2610	0	0	0	2720	23012
B	0	1577.9	0	0	0	2.1	0
C	1850	2975	0	2448.95	0	2826.05	1850
D	11040	3165	0	0	0	6605	11040
E	0	0	0	2110	0	0	0
F	550	5455.1	774.36	4795.21	0	18025.33	550
G	6680	4165	0	0	0	13605	6680
H	1340	3765	0	5303.2	0	5501.8	1340
I	0	1904.28	0	1768.09	0	10721.63	0
J	0	1840	0	0	0	23210	0
K	0	0	0	0	0	32250	0
L	0	0	0	0	0	19530	0

Table 14. Efficiency decomposition of DMUs in real example when input reduction is preferred.

DMU	Input Congestion EFF.	Input Tech. EFF.	Price Congestion EFF.	Price Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	1	1	0.83	0.88	1	0.85	0.62
B	1	1	1	0.91	1	0.99	0.91
C	1	0.88	0.93	0.88	1	0.85	0.61
D	1	1	0.70	0.88	1	0.70	0.43
E	1	0.88	1	1	1	1	0.88
F	0.98	0.87	0.99	0.88	1	0.46	0.34
G	1	1	0.83	0.88	1	0.53	0.39
H	1	0.80	0.96	0.88	1	0.74	0.49
I	1	0.94	1	0.94	1	0.59	0.52
J	1	1	1	0.95	1	0.40	0.38
K	1	1	1	1	1	0.33	0.33
L	1	1	1	1	1	0.44	0.44

Table 15. Efficiency decomposition of DMUs in real example when price reduction is preferred.

DMU	Price Congestion EFF.	Price Tech. EFF.	Input Congestion EFF.	Input Tech. EFF.	Mixed EFF.	Allocative EFF.	Overall EFF.
A	0.83	0.88	1	1	1	0.85	0.62
B	1	0.91	1	1	1	0.99	0.91
C	0.93	0.88	1	0.88	1	0.85	0.61
D	0.70	0.88	1	1	1	0.70	0.43
E	1	1	1	0.88	1	1	0.88
F	0.99	0.88	0.98	0.88	1	0.46	0.34
G	0.83	0.88	1	1	1	0.53	0.39
H	0.96	0.88	1	0.80	1	0.74	0.49
I	1	0.94	1	0.94	1	0.59	0.52
J	1	0.95	1	1	1	0.40	0.38
K	1	1	1	1	1	0.33	0.33
L	1	1	1	1	1	0.44	0.44

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5 | Conclusion

One of the main strengths of DEA compared to other methods of evaluating the performance of DMUs, is that in addition to providing a performance score, it also provides factors affecting inefficiencies, so unit decision makers understand the reasons for the weakness and can plan to fix them. If, in addition to the input and output data of DMUs, price information is also available, various factors influencing cost inefficiency can be determined. If the prices for the units are not the same, it is possible that the units have been inefficient in providing resources at a reasonable price. Therefore, in addition to inefficiency factors related to the quantities of inputs and outputs, inefficiency factors in prices must also be determined. In this paper, for the first time, three PPS was used to provide a decomposition for CE. Accordingly, congestion and technical inefficiencies in the PPS based on input and output quantities (P_x), and based on input and output prices (P_c) were considered. In addition, two other types of inefficiencies, mix and allocative inefficiencies, were introduced in the third PPS (P_{cx}), which was the result of input and optimal price vectors resulting from operations in the first two sets. Since each type of cost inefficiency leads to an excess cost, we express the cost of observation as the sum of the optimal cost and excess costs. Based on whether in the process of fixing inefficiencies, fixing inefficiencies related to inputs is a priority or fixing price inefficiencies, we presented two types of analysis for CE as well as observed cost.

In the present study, the proposed efficiency decomposition is apt for a case in which the input price data are available. If the output prices, in addition to the input price data, are available, one can analyze the profit and income efficiencies by developing the proposed method. Due to the existence of inaccurate and ambiguous data in the amount of inputs and costs, and the importance of fuzzy DEA, the researchers are refered to [27]–[32] for future studies. Moreover, the proposed method can be used when the structure of the DMUs is in the form of a network. In the evaluation, Moreover, the units were considered to be quite independent, and there was not any discussion concerning the hierarchical and group structures, not for a case in which the units are under a centralized or original decision-maker with the possibility of resource displacements, which can be considered in future studies.

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Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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A Multi-Objective and Multi-Level Model for Location-Routing Problem in the Supply Chain Based on the Customer's Time Window

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Abstract

Today, logistics costs often make up a major part of large organizations' expenses. These costs can be reduced with optimal design and its implementation in the supply chain. As a result, in present study, a two-objective mathematical location-routing model is presented, where an objective is to minimize the costs and the next is to maximize the reliability in order to deliver the goods timely to customer according to the probable time and time window. The proposed problem has two levels of distribution. The first level, which is called transportation level, points to the distribution of products from a factory to an open distribution center, and the latter is known as routing level, which is related to a part of the problem in which we deliver products from the warehouse to customers. The proposed mathematical model is solved by Epsilon-constraint and NSGA-II approaches in small and medium, and large scales problem, respectively. The present study has provided the following contributions: concurrent locating and routing in the supply chain in accordance with the customer's time window, probable travel time in the supply chain and customer's reliability in the supply chain. The assessment metric results indicate the proper performance of our proposed model.

Keywords: Location-routing problem, Reliability, Time window, Meta-heuristic algorithm.

1 | Introduction

Over the last years, efficient, reliable, and flexible decisions on locating the warehouses and routing the distribution paths have been of high importance to managers [1], [2]. Many research report that if routing is neglected while warehouses are located, the cost of distribution systems may increase [3], [4]. The Location-Routing Problem (LRP) has overcome this problem by concurrent consideration of location and routing [5], [6]. LRP is used in many fields such as food distribution, newspaper delivery, wastes collection, drug distribution, military applications, postal parcels delivery, natural disaster relief goods distributionn, and distribution of various goods among customers [7]–[9].

In the classical models of LRP problems, a number of nodes are deployed as potential locations for warehouses and some nodes are considered as customers in specific locations [10], [11].

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In these problems, limited capacity is generally considered for warehouses and vehicles which is called Capacitated Location Routing Problem (CLRP) [12]–[14]. One answer to this question is regarding the open warehouses, each customer should be allocated to an active warehouse, and routes should be created for each warehouse and the corresponding customers [15], [16]. The following limitations should be taken into account: 1) the total demand of customers allocated to a warehouse should not exceed the capacity of the warehouse, 2) each route starts from a warehouse and ends at the same warehouse, 3) each vehicle makes a maximum of one trip, 4) each customer is served by only one vehicle (no separate delivery), and 5) The total demand of customers who are met by one vehicle should be proportional to the capacity of the vehicle [17], [18]. This two-step LRP includes three sub-problems: Facility Location Problem (FLP), products transportation from factories to warehouse (a transportation proportional to the truck capacity), and vehicle route design in order to meet customers of any intracity distribution center, referred to as the Transportation Location-Routing Problem (TLRP) [19]–[22].

Due to traffic jam and road problems, time never can be certainly said as fixed, because in each route from the warehouse to the customer or between customers, the travel time of the vehicle is a probable variable with a uniform distribution [23]. Customers' time window is embedded for timely product delivery. Maintaining efficiency for logistics distribution not only reduces the procurement costs, but is also very important in the quality of services [4]. Given the time window, customer demand must be met in time. If a vehicle arrives at the customer's place earlier than the deadline, it incurs the waiting cost, and if it is delayed, the customer's satisfaction decreases or the contract may even be canceled in strict circumstances. These additional costs of early and delay arrivals are called penalty costs. For this reason, by combining the time window and TLRP, the problem gets closer to the real world. Adding reliability to the model is essential to ensure the level of service to each customer node. By reliability modeling, the method presented in [24], was approved and developed. Reliability of service-delivery is considered as an indicator of quality for transportation, which has been highly regarded in recent years. The variety of real-time travel per vehicle (as a variety of service) affects the reliability of service and travel time of the passenger. Hence, we have developed a transportation location-routing model along with a probable travel time and a time window with two objectives including costs minimization and the maximization of customer's reliability on timely delivery, attracting the customers' complete satisfaction. The contributions of papers are as follows:

- *Paying attention to location and routing in the supply chain and the customer's time window.*
- *Paying attention to the probable travel time in the supply chain.*
- *Paying attention to customer's reliability.*

This paper is organized as follows; the introduction is presented in Section 1. In Section 2, the literature and the background of the problem will be reviewed; followed by these two sections, three proposed mathematical models are presented in Section 3; the problem solving methods and the results of the calculations are mentioned in the Section 4 and Section 5; and finally the conclusions will be presented in the Section 6.

2 | Literature Review

The first studies on LRP date back to the 1970s and early 1980s. Of the first authors analyzing LRP were Watson-Gandy and Dohrn [25], and to date, many studies have been conducted in this field. Few works were found on multi-stage LRP. In Tirkolaee et al. [26], a four-layer LRP model is designed. The optimal solution for a sample was obtained after a limited processing time of 25 hours for small samples and several days for large samples. In Bashiri et al. [27], a two-stage LRP model is presented. The authors divided the problem into two modules including location/allocation and routing, getting good results for small samples but having very time-consuming calculations.

Panicker et al. [28] presented a two-level model for routing and locating distribution centers in conditions of uncertainty. Considering replenishment at intermediate depots is one of the innovations of their research. Also, they examined the time window as well as loading and docking time for transporting the goods. Finally, due to the NP-hard nature of the model, the ant colony optimization algorithm was used to solve the problem.

Balcik [29] provided a mathematical model for locating distribution centers and making routing decisions at uncertainty conditions in the Turkish city Van as the case study. His proposed model aimed to cover all considered areas so that the coverage rate would be maximized. This model is solved with the Tabu search algorithm.

Herazo-Padilla et al. [30] for the first time increased the complexity of LRP by simultaneously considering the assumptions of multi-period, multi-product, and uncertainty of demand. Their three-tier supply chain consists of one supplier, several warehouses and customers. In this model, customers and warehouses use a periodic review policy to replenish the inventory. Therefore, inventory shortage and maintenance costs are included in the objective function for both customers and warehouses. Furthermore, customer demand is uncertain and follows Poisson distribution.

Ghasemi et al. [31] presented a multi-objective, multi-product and multi-period mathematical model for managing crisis relief decisions in Tehran as their case study. The decisions considered in this study included location, allocation of ambulances and investigation of the flow of relief goods. Consideration of the failure of distribution centers was one of the innovations for this research. The proposed mathematical model is solved using NSGA-II and MMOPSO approaches, indicating the proper performance of their proposed model.

In Zeng et al. [32], a variable and probabilistic time is included in Vehicle Routing Problem (VRP) schedules where clients do not have a time window. The model was developed by extending the classical VRP objective function and considering the expected time and standard travel time deviation. In Ganesh et al. [33], the VRP model was developed by considering variable and probabilistic times as well as soft time windows with the aim of minimizing shipping and the fines costs.

By considering Tehran as their case study, Ghasemi and Khalili-Damghani [34] proposed a robust mathematical model for locating distribution centers, controlling the inventory of relief goods, and allocating centers to hospitals in earthquake conditions. Estimating the demand for relief goods with the help of computer simulation and interactive design of basic urban infrastructure are of their innovations. The simulation-optimization model is solved using robust optimization. The findings indicated the proper performance of their proposed model. Gerdrodbari et al. [35] presented a multi-level, multi-period, multi-product Closed-Loop Supply Chain (CLSC) for timely production and distribution of perishable products. To solve the model, the robust optimization method is utilized. To validate the model in small-size problems, the epsilon-constraint method was presented and non-dominated sorting genetic algorithm was developed for solving large-size problems. Son et al. [36] developed a mathematical model for managing a robust CLSC. Minimizing environmental costs and transportation costs along with maximizing social impacts are among the innovations of their research. To this end, two mathematical models including a robust model and an evaluation model were proposed to minimize the system costs. Due to the NP-hard nature of the model, a genetic approach was to solve the model, indicating the proper performance of their proposed model in minimizing the costs. In Florio et al. [37], the cost of unpredicted fines for planning each tour is presented to reflect the costs incurred in actual operations to adjust the arrival sooner or later than the scheduled time in LRP. In Araghi et al. [38], LRP has been extended in large-scale which was formulated as a multi-commodity integer network flow problem.

Also, a complete and practical location-routing model is proposed in a distribution network to improve customer satisfaction [39]. A Fleet Size and Mixed Location-Routing Problem with Time Windows (FSMLRPTW) problem is proposed by taking into account the number of vehicles and the different time window that aims to minimize fixed vehicle and warehouse costs as well as routing costs [40].

In Ebrahim Qazvini et al. [41], reliability is defined as the lower limit of the probability that if an accident occurs in one of the demand nodes, it will be handled immediately by a vehicle located in one of the warehouses. Accordingly, the desired level of reliability can be achieved by limiting the probability of failure. Furthermore, a framework is presented along with cost-effective design tools to develop the level of reliability of service from a passenger's perspective. Service reliability is known as one of the indicators based on the schedule, which is perceived by the user and is one of the most important indicators of public transport quality. *Table 1* summarized the literature as follows:

Table 1. Summary of literature.

Ref.	Number of Objective Functions (Single/Multi Objective-(SO)/(MO))	Type of Objective Functions (Cost (C)/ Reliability (R))	Echelon (Multi (M), Single(S))	Time Windows	Decision Types (Allocation (Al) / Network Flow (F)/ Routing (R))				Meta-Heuristic (M), Exact Algorithm (E)
					Loc	Route	Al	F	
[42]	SO	R	M			*			E
[40]	SO	C	S	*	*	*			E
[38]	MO	C	M		*	*			M
[37]	SO	C	S		*	*			E
[36]	MO	C	M				*		M
[34]	MO	C	M		*				E
[26]	MO	C	M		*				M
[27]	MO	C	M		*	*	*		M
[33]	SO	C	S	*		*			E
[32]	SO	C	S	*		*			E
[31]	MO	C	M	*		*			M
[29]	SO	C	S		*	*			E
This Study	MO	CR	M	*	*	*	*	*	EM

Therefore, according to the literature review, the research gap is described as follows:

- *Not concurrent paying attention to location and routing in the supply chain and the customer's time window.*
- *Not paying attention to the probable travel time in the supply chain.*
- *Not paying attention to customer's reliability in the supply chain.*

3 | The Problem Definition

The customers have a definite demand. Their product demand is supplied from a range of factories. Each customer j has a specific demand b_j and each factory f can supply a maximum of a_f units of the product. On the other hand, the factory cannot deliver the products directly to the customer; for this reason, the warehouse acts as an intermediate point so that the products are sent from the factory to these centers and these centers deliver the products to the customer. This is a classic situation for the urban logistics problem, where trucks carry the final products from the factory but they cannot deliver the products to customers located in the city. Therefore, they carry the goods to the warehouses, which are usually located in the suburbs, and unload the products.

Warehouse candidate centers are places where warehouse facilities are located or places where warehouses can be built. Each warehouse i has a reopening cost of g_i and a capacity of b_i. In general, not all candidate warehouses need to be reopened and used, and only a few will be employed. *Fig. 1* shows the framework of proposed supply chain.

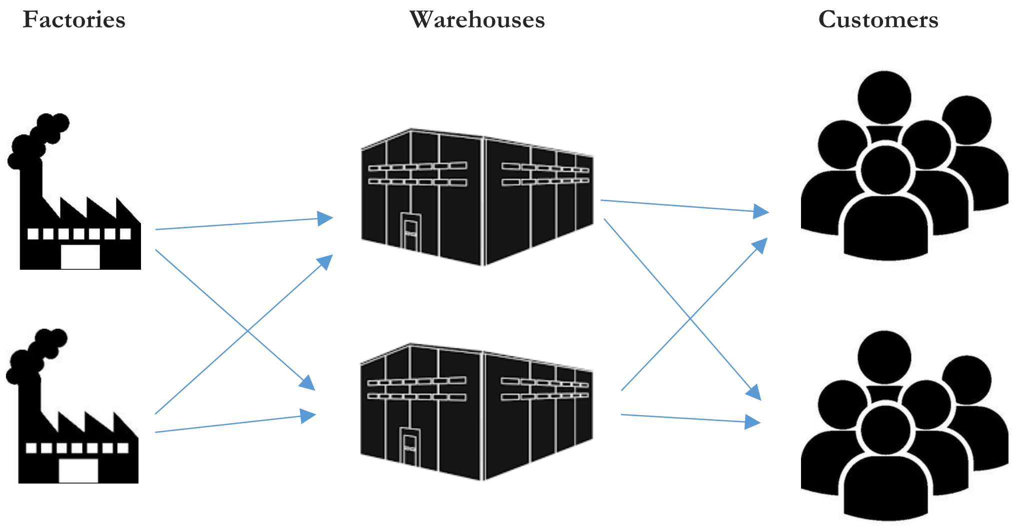

Fig. 1. Supply chain framework.

The problem has two levels of distribution, which is a part of linked *FLP*. The first level, which is called transportation level, points to the distribution of products from a factory to an open distribution center, and the latter is routing level, which is the part of the problem that we deliver products from the warehouse to customers. In the second objective, we look at the problem differently. For competition, service providers need to deliver the right level of customer service, which often depends on the choice of centers, the distance of customers from the warehouse, the reliability of shipping the requested products to support the customer in a given time, and the relationship between shipping routes.

3.1 | Assumptions

Specifications of level 1 of the objective 1:

- *There is a number of* f *factories for production.*
- *It includes only one type of product.*
- *Each factory* f *can supply a maximum of* a_f *products.*
- *There is a set of* m *candidate sites where the warehouse can be located. Each candidate site has a capacity of* b_i *and a fixed cost of* g_i *to stay open.*
- *The final product must be shipped from the factory to open warehouse. Transportation at this level is done by trucks. Each truck has a capacity* CR *and a cost* d_{fi} *to transport from the factory to the warehouse.*

Specifications of level 2 of the objective 1:

- *There is a set of* n *customers that each customer* j *has a demand* b_j.
- *From every warehouse in operation, products are shipped for customers by vehicles. Delivery of products is done through a series of routes. Vehicles must leave the warehouse, meet the customer on the way and return to the warehouse.*
- *Each vehicle has a limited loading of* Q *units of the product.*
- *There is a cost* c_{ij} *which points to the distance between warehouse and customer pr between the two customers.*
- *The vehicles are considered same.*
- *A set of* k *vehicles delivers the products to the customers at the level of the route and returns to the warehouse.*
- *Travel time from the warehouse to the customer or between customers is probabilistic.*

Specifications of objective 2:

- *A time window has been considered for timely delivery to customers.*
- *Possible reliability function is uniform with the distribution function.*
- *Uncertainty should not be lower than* β.

A problem has been formulated with two objectives, cost minimization and maximizing customer service assurance at the right time. We defined a mathematical model for this two-objective problem.

Sets

p	Number of factories.
i	Number of candidate sites for routing the warehouses.
N	Number of customers.
$f=\{1,2,...,p\}$	Set of indicators for factories.
$D=\{1,2,...,m\}$	Set of indicators for warehouses.
$J=\{m+1,m+2,...,m+n\}$	Set of indicators for customers.

Parameters

a_f	The product capacity in the factory f.
d_{fi}	The cost of shipping a truck from the factory f to the warehouse i.
CR	Capacity of truck, factory f, warehouse i.
g_i	The fixed opening and operating cost of the warehouse i.
b_i	Capacity of the warehouse i.
h_i	Demand of the customer j.
c_{ij}	Customer meeting cost/ warehouse i exactly after customer/distribution center j.
Q	Capacity of the vehicle in the route level.
d_{fi}	The cost of travel from the factory f to the warehouse i.
s_j	The time of arriving to the customer j.
cl_j	Cost of fines per unit of late arrival to the customer j.
$Cwait_j$	Cost of fines per unit of early arrival to the customer j.
β	Minimum reliability.

Variables

V_{fi}	The amount of shipped products from factory f to the warehouse i.
W_{fi}	Number of transformed trucks from the factory f to the warehouse i.
e_j	Time of leaving customer j.
ar_j	Time of arriving to the customer j.
$wait_j$	Waiting time of customer j (free in sign).

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$$y_i = \begin{cases} 1, & \text{In case of open warehouse } i, \\ 0, & \text{Otherwise.} \end{cases}$$

$$z_{ij} = \begin{cases} 1, & \text{In case of assigning customer } j \text{ to the warehouse } i, \\ 0\ , & \text{Otherwise.} \end{cases}$$

$$o_{jk} = \begin{cases} 1, & \text{In case of assigning vehicle } k \text{ to the customer } j, \\ 0, & \text{Otherwise.} \end{cases}$$

$$x_{ijk} = \begin{cases} 1, & \text{In case of meeting customer or warehouse } j \text{ afetr customer} \\ & \text{or warehouse } i \text{ by the vehicle } k, \\ 0, & \text{Otherwise.} \end{cases}$$

$$\begin{aligned} \operatorname{Min} f_1 = {} & \sum_{\forall f \in F} \sum_{\forall i \in D} d_{fi} w_{fi} + \sum_{\forall i \in D} g_i y_i + \sum_{\forall j \in J} \sum_{\substack{\forall i \in D \\ j \neq i}} \sum_{\forall k \in k} ff_i x_{ijk} + \\ & \sum_{\forall j \in J} \sum_{\substack{\forall i \in v0 \\ j \neq i}} \sum_{\forall k \in k} c_{ij} x_{ijk} + \sum_{\forall j \in J} cl_j late_j + \sum_{\forall j \in J} Cwait_j wait_j . \end{aligned} \tag{1}$$

$$\operatorname{Max} f_2 = \prod_{\text{for all } i \in v0} \prod_{\substack{\text{for all } j \in J \\ j \neq i}} \prod_{\text{for all } k \in k} R_j^{x_{ijk}} . \tag{2}$$

$$\sum_{\text{for all } i \in D} v_{fi} \leq a_f \quad \text{for all } f \in F. \tag{3}$$

$$w_{fi} \geq \frac{v_{fi}}{CR} \quad \text{for all } f \in F, \text{ for all } i \in D. \tag{4}$$

$$\sum_{\text{for all } f \in F} v_{fi} \leq b_i y_i \quad \text{for all } i \in D. \tag{5}$$

$$\sum_{\text{for all } f \in F} v_{fi} = \sum_{\text{for all } j \in J} h_j z_{ij} \text{ for all } i \in D. \tag{6}$$

$$\sum_{\text{for all } i \in D} z_{ij} = 1 \quad \text{for all } j \in J. \tag{7}$$

$$\sum_{\text{for all } i \in D} \sum_{\text{for all } k \in k} x_{ijk} = 1 \quad \text{for all } j \in J. \tag{8}$$

$$\sum_{\text{for all } i \in D} \sum_{\text{for all } j \in J} x_{ijk} \leq 1 \quad \text{for all } k \in k. \tag{9}$$

$$\sum_{\text{for all } m \in J} x_{imk} + \sum_{\text{for all } h \in J} x_{jhk} = 1 + z_{ij} \quad \text{for all } i \in D, \text{ for all } j \in J, \text{ for all } k \in k. \tag{10}$$

$$\sum_{\text{for all } i \in v0} x_{ijk} = \sum_{\text{for all } i \in v0} x_{jik} \quad \text{for all } j \in v0, i \neq j, \quad \text{for all } k \in k. \tag{11}$$

$$u_i - u_j + Q \sum_{\text{for all } k \in k} x_{jik} \leq Q - h_j \quad \text{for all } i, j \in J, i \neq j. \tag{12}$$

$$h_j \leq u_j \leq Q \quad \text{for all } j \in J. \tag{13}$$

$$\sum_{\text{for all } i \in v0} x_{ijk} = o_{jk} \quad \text{for all } k \in k, \quad \text{for all } j \in J, \ i \neq j. \tag{14}$$

$$\sum_{\text{for all } j \in J} x_{ijk} = o_{jk} \quad \text{for all } k \in k, \quad \text{for all } i \in v0, \ i \neq j. \tag{15}$$

$$10 \leq T_{ij} \leq 20 \quad \text{for all } i, j \in v0, i \neq j. \tag{16}$$

$$e_i = 0 \quad \text{for all } i \in D. \tag{17}$$

$$\sum_{\text{for all } j \in J} ar_j x_{ijk} = \sum_{\text{for all } j \in J} T_{ij} x_{ijk} \quad \text{for all } i \in D, i \neq j, \quad \text{for all } k \in k. \tag{18}$$

$$\sum_{\text{for all } j \in J} ar_j x_{ijk} = \sum_{\text{for all } j \in J} \left(ar_i + s_i + T_{ij}\right) x_{ijk} \quad \text{for all } i \in J, i \neq j, \quad \text{for all } k \in k. \tag{19}$$

$$\left(ar_j - s_i - ar_i - T_{ij}\right) \leq M\left(1 - x_{ijk}\right) \quad \text{for all } i, j \in v0, i \neq j, \quad \text{for all } k \in k. \tag{20}$$

$$\sum_{\text{for all } i \in v0} down_j x_{ijk} \leq \sum_{\text{for all } i \in v0} ar_j x_{ijk} \leq \sum_{\text{for all } i \in v0} up_j x_{ijk} \quad \text{for all } j \in J, i \neq j \quad \text{for all } k \in k. \tag{21}$$

$$\sum_{\text{for all } i \in v0} wait_j x_{ijk} = \sum_{\text{for all } i \in v0} (ar_j - down_j) x_{ijk} \quad \text{for all } j \in J, i \neq j, \quad \text{for all } k \in k. \tag{22}$$

$$waitt_j = \max\left\{0, wait_j\right\} \quad \text{for all } j \in J. \tag{23}$$

$$\sum_{\text{for all } i \in v0} late_j x_{ijk} = \sum_{\text{for all } i \in v0} (ar_j - up_j) x_{ijk} \quad \text{for all } j \in J, i \neq j, \quad \text{for all } k \in k. \tag{24}$$

$$latee_j = \max\left\{0, late_j\right\} \quad \text{for all } j \in J. \tag{25}$$

$$R_j = \frac{(up_j - ar_j)}{(up_j - down_j)} \quad \text{for all } j \in J. \tag{26}$$

$$\prod_{\text{for all } j \in J} \frac{1}{(up_j - ar_j)} \leq 1 - \beta. \tag{27}$$

$$0 \leq R_j \leq 1 \quad \text{for all } j \in J. \tag{28}$$

$$y_i \in \left\{0,1\right\} \quad \text{for all } i \in D. \tag{29}$$

$$z_{ij} \in \left\{0,1\right\} \quad \text{for all } i \in I, \quad \text{for all } j \in J. \tag{30}$$

$$o_{jk} \in \left\{0,1\right\} \quad \text{for all } k \in K, \quad \text{for all } j \in J. \tag{31}$$

$$x_{ijk} \in \left\{0,1\right\} \quad \text{for all } i, j \in v\,0, i \neq j, \quad \text{for all } k \in K. \tag{32}$$

$$v_{fi} \geq 0; w_{fi} \in Z_+ \quad \text{for all } f \in F, \quad \text{for all } i \in D. \tag{33}$$

$$e_j \geq 0; ar_j \geq 0 \quad \text{for all } j \in J. \tag{34}$$

$$wait_j \in Z; late_j \in Z \quad \text{for all } j \in J. \tag{35}$$

$$waitt_j \geq 0; latee_j \geq 0 \quad \text{for all } j \in J. \tag{36}$$

Objective *Function (1)* points to the economic objective including the fixed and operating costs of reopening the warehouse and operating a vehicle at candidate site i, the varying costs including transporting from the factory to the warehouse and then to the customer, and the costs of fines for early or late delivery of products to the customer. Objective *Function (2)* calculates reliability over time at the route level.

Constraint (3) ensures that the product is shipped not more than the factory capacity. *Constraint (4)* expresses transforming trucks from any factory to any warehouse according to the capacity of the trucks. The quantity of the product shipped from each factory to each warehouse should not exceed the warehouse capacity, according to *Constraint (5)*, if $y_i=0$, then no product will be shipped to the warehouse *i*. In accordance with *Constraint (6)*, the quantity of products shipped to a warehouse must be equal to the total demand of the customers allocated to that warehouse. *Constraint (7)* assures us that each customer is allocated to only one warehouse. The *Constraints (8)* to *(10)* first ensure that each customer must be met immediately after a warehouse or after other customers. Second, the structure of the routes takes place between the customers assigned to the common warehouse. According to the *Constraint (11)*, the number of vehicles leaving the warehouse should be equal to the number of vehicles returning to the warehouse. *Constraints (12)* and *(13)* are relevant to the sub-tours, so that the capacity of vehicles while traveling between two customers should not be exceeded. *Constraints (14)* and *(15)* state allocation of the vehicle to each customer. The *Constraint (16)* expresses that the travel time is a probable variable. Given the *Constraints (17)* to *(19)*, the time of departure from the warehouse should be the beginning of work and in fact equal to zero, expressing the time of departure of the first and the rest of customer. *Constraint (20)* and *(21)* calculate the soft time window. *Constraints (22)* to *(25)* calculate the amount of waiting and delay times. *Constraints (26)* to *(28)* express the reliability from the time of delivering services to the customers. *Constraints (29)* to *(36)* specify the type of decision variables.

4 | Problem Solving Method

Epsilon constraint

Epsilon constraint is one of the most accurate methods for obtaining Pareto optimal solutions. This method is one of the well-known approaches to dealing with multi-objective problems, which solves these types of problems by transferring all but one of the objective functions to the constraints at each stage. The Pareto boundary can be created by the ε-constraint method as the following equation [42]:

$$\begin{aligned} &\min f_1(x), \\ &\quad x \in X, \\ &\quad f_2(x) \le \varepsilon_2, \\ &\quad \vdots \\ &\quad f_n(x) \le \varepsilon_n. \end{aligned} \tag{37}$$

The steps of this method are as follows:

I. Select one of the objective functions as the main objective function, solve the problem each time according to one of the objective functions and obtain the optimal value of each objective function.
II. Divide the interval between the two optimal values of the sub-objective functions into a predetermined number and obtain a table of values for $\varepsilon_2, \dots, \varepsilon_n$.
III. Solve the problem each time by the main objective function with any of the values $\varepsilon_2, \dots, \varepsilon_n$.
IV. Report the found Pareto solutions.

NSGA-II method

Non-dominated Sorting Genetic Algorithm II (NSGA-II) is one of the most popular and widely used optimization algorithms in the field of multi-objective optimization. This algorithm is introduced by Deb [43]. This algorithm uses only the values of the objective function to perform the optimization process and does not require any additional information such as the derivative of the function. It is also very fast and efficient due to the simplicity of the search process. Solving the problem by this algorithm requires a number of elements, one of which is the archive of reasonable answers [43]. On the other hand, because the computer memory is limited, the number of archives cannot be allowed to grow as much as it desires. As a result, an archive size control mechanism is needed to control the number of archived responses. If these two features are combined with the search feature, an intelligent optimization algorithm will be obtained. For example, *Fig. 2* shows the chromosome corresponding to the variable $o_{j,k}$. If vehicle k is allocated to the customer j, the amount of gene will be 1, otherwise it will be zero.

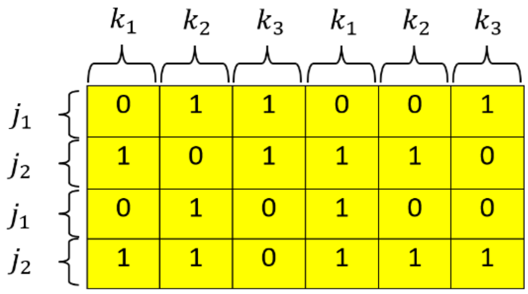

Fig. 2. The chromosome corresponding to the variable $o_{j,k}$.

The main operators of this method are mutation and cross-over, which are defined in this study as follows: *Fig. 3* shows the cross-over operator. Double-point type of crossover was used in this study. The mechanism of this operator is in such a way that two points are randomly selected and the strings of each chromosome will be displaces.

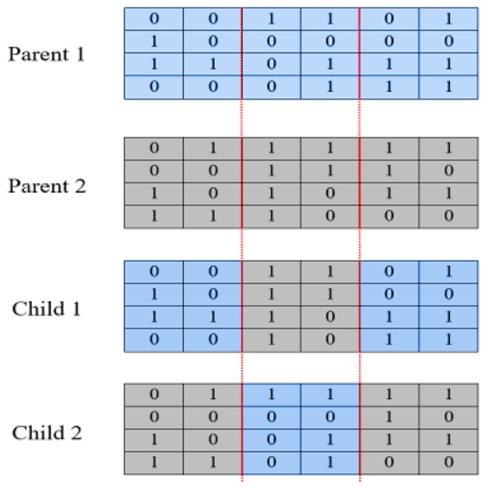

Fig. 3. Double-point crossover.

Fig. 4 shows the mutation operator. For this purpose, a row is selected optionally and the selected row will be reversed.

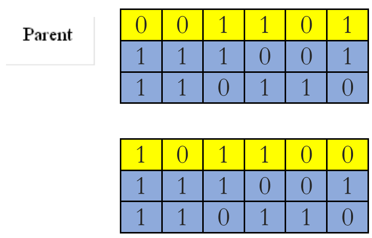

Fig. 4. Mutation operator.

4.1 | Assessment Metric

Space Metric (SM)

This metric measures the uniformity of NSGA-II solutions. It is calculated using the following equation:

$$SM = \frac{\sum_{i=1}^{n-1}\left|d_i - \bar{d}\right|}{(n-1)\bar{d}}. \quad (38)$$

In this equation, d_i is the Euclidean distance between two adjacent Pareto points and $\bar{d}$ is the mean Euclidean distances. Therefore, the closer the SM value to zero, the equal the distance between adjacent Pareto points.

The Mean Distance from the Ideal point (MID)

This metric is a qualitative indicator and calculates the distance between the Pareto edges and the ideal point. Undoubtedly, the lower the value of this indicator, the better the quality of the answer. It is calculated as follows:

$$MID = \frac{\sum_{i}^{n}\sqrt{\left(\frac{f_{1i} - f_1^{best}}{f_{1,total}^{max} - f_{1,total}^{min}}\right)^2 + \left(\frac{f_{2i} - f_2^{best}}{f_{2,total}^{max} - f_{2,total}^{min}}\right)^2 +}}{n}, \quad (39)$$

where n is the number of Pareto points, f_{1i}^2 and f_{2i}^2 are the values of the first and second targets for the i^{th} Pareto answer, respectively. Also, $f_{j,total}^{max}$ and $f_{j,total}^{min}$ are the maximum and minimum values of the j^{th} objective function between Pareto points, respectively. According to this definition, a lower MID value indicates better performance of the algorithm.

Table 2 shows the results of solving small and medium problems. The first five samples are Pareto points of small problems and the next five ones are those of medium size problems. The columns 2 to 4 are the values of the first and second objective functions as well as the solution time by the Epsilon constraint method, and the columns 5 to 7 are the first and second objective functions, as well as the solution time by NSGA-II method. The last two columns are the amount of error due to NSGA-II. As can be seen, the mean error for both objective functions is lower than one percent.

Table 2. The results of solving the problems by Epsilon-Constraint and NSGA-II methods.

No.	Epsilon-Constraint					NSGA-II				
	Function 1	Function 2	SM	MID	Time (s)	Function 1	Function 2	SM	MID	Time (s)
1	128.5	4.84	0	6.45	2	128.5	4.84	0	6.45	2
2	131.7	4.48	0	6.48	26	131.8	4.48	0	6.48	6
3	132.2	4.09	0.08	6.55	52	135.8	3.97	0.10	6.68	7
4	132.8	3.71	0.09	6.60	93	136.1	3.66	0.12	6.69	9
5	133.9	2.77	0.15	6.71	142	137.7	2.72	0.18	6.73	14
6	2362.1	8.38	0.13	6.64	1351	2371.5	8.34	0.16	6.68	23
7	2379.6	7.51	0.22	6.73	2875	2386.4	7.33	0.26	6.77	26
8	2418.7	7.16	0.14	6.70	3742	2426.0	6.99	0.19	6.75	32
9	2437.4	6.55	0.17	6.78	4981	2443.2	6.44	0.21	6.84	38
10	2561.2	6.39	0.15	6.83	6412	2572.9	6.27	0.20	6.87	41
AVE	1409.9	5.58	0.113	6.647	1967.6	1286.99	5.504	0.14	6.69	19.8

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5 | Numerical Results

In this paper, an example was generated to evaluate the proposed model, which is solved by Epsilon constraint and NSGA-II methods in the platform of LINGO ver.9.0 and MATLAB software packages. Preliminary data on this problem include the cost of travel between nodes, the cost of using the vehicle, the time of delivering services to customer, the cost of penalty for delayed arrival and the cost of penalty for early arrival which have the values 10, 12, 5, 30 and 10, respectively. The capacity of the vehicle is equal to 160 and that of the truck is equal to 220. The amount of factories production of the desired product is 800 and 600 and the rest of the data is given in *Table 3*.

Table 3. The problem nodes-related parameters.

Nodes Qty. Parameters	1	2	3	4	5	6	7	8
Customers demand	0	0	0	40	50	80	60	50
Warehouse reopening cost	6	5	6	0	0	0	0	0
Warehouse capacity	240	220	180	0	0	0	0	0
Lower limit of the customer's time window	0	0	0	40	25	10	10	25
Upper limit of the customer's time window	0	0	0	53	38	20	20	40

Table 4 and *Figure 5* present the results of Pareto points' extraction. As it turns out, 5 Pareto points have been extracted. For example, the third one is equal to (134, 0.8).

Table 4. Extracted Pareto points for both objective functions.

Objective Function 1 (Cost)	Objective Function 2 (Reliability)
105	1
125	0.9
134	0.8
149	0.75
160	0.65

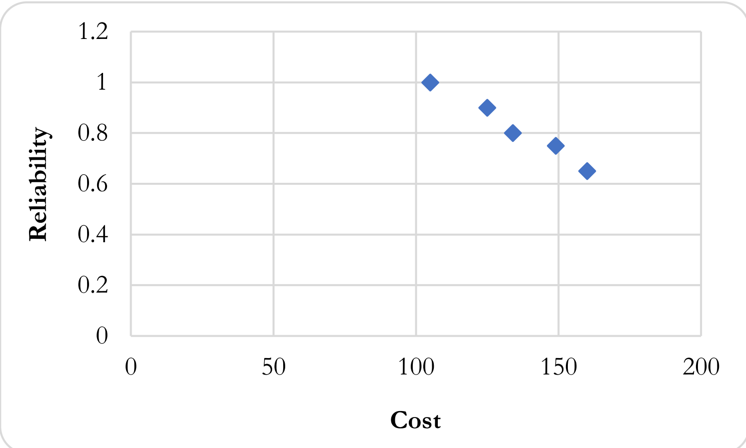

Fig. 5. The set of Pareto points.

6 | Conclusion

One of the most important issues in logistics networks is the design and analysis of distribution networks. In recent years, two main issues in the design of distribution networks, namely the location of warehouse and routing of distributors with each other have been considered and have created the LRP. In this paper, we presented a nonlinear mixed integer programming model for the TLRP by considering the probable travel time and the time window. This model aims to achieve two objectives, namely costs minimization and the customer reliability maximization which includes timely delivery to customers and their complete satisfaction.

The proposed mathematical model is developed in small and medium dimensions by epsilon constraint approach and in large dimensions by NSGA-II algorithm. Among the innovations considered in this research, we may refer to paying attention concurrently to location and routing in the supply chain with the customer's time window, paying attention to the probable travel time in the supply chain and considering the customer's reliability in the supply chain. The assessment metric results indicate the proper performance of our proposed model. The results of this research can be useful for the food industry, all medicine distribution companies, food companies, etc. Managers can also consider strategic decisions such as location in the form of sustainable development. Managers can also dynamically consider operational decisions such as routing to minimize supply chain costs. Considering the time window of customers can also increase customer satisfaction, which allows managers to increase supply chain profits. The research results are as follows:

- *The cost of travel between nodes, the cost of using the vehicle, the time of delivering services to customer, the cost of penalty for delayed arrival and the cost of penalty for early arrival which have the values 10, 12, 5, 30 and 10, respectively.*
- *The capacity of the vehicle is equal to 160 and that of the truck is equal to 220.*
- *The amount of factories production of the desired product is 800 and 600.*

In this study, as there was no systematic database for some parts of transportation cost elements, driver's estimations and transportation officers were asked to help.

The following items are proposed for the future studies:

- *Considering a competitive game between supply chain members in the proposed model.*
- *Considering other objectives in the model such as maximizing the resilience of supply chain or minimizing the delivery time.*
- *Considering uncertainties in prices and problem solving with a two-tier planning approach.*
- *Consider multiple transportation models, such as lorry, container/trailer or rail and air transportation.*

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Introducing a Fuzzy Robust Integrated Model for Optimizing Humanitarian Supply Chain Processes

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Abstract

The natural disasters of the last few decades clearly reveal that natural disasters impose high financial and human costs on governments and communities. Concerns in this regard are growing day by day. Making the right decisions and taking appropriate and timely measures in each phase of the crisis management cycle will reduce potential damage at the time of the disaster and reduce the vulnerability of society. Therefore, in this research, a mathematical model of crisis logistics planning considering the problem of primary and secondary crisis in disaster relief is introduced, which is the innovation of this research. In the primary crisis, the goal is to provide services and relief goods to crisis areas, and in the second stage, the secondary crisis that occurs after the primary crisis seeks to provide relief to crisis centers and transfer the injured to relief centers. Therefore, this research proposes a mathematical fuzzy ideal programming model in two primary and secondary crises. In the primary crisis, the goal is to provide services and relief goods to crisis-stricken areas. The secondary crisis, which occurs after the primary crisis, aims to support crisis-stricken centers and move injured people to relief bases in the second step. According to the proposed model, Bertsimas-Sim's fuzzy programming that formulation proposed by Bertsimas and Sim [1] and robust approach we initially used. The Epsilon constraint method was used to solve the low-dimensional model. Multi-objective meta-heuristic algorithms have been designed to handle the computational complexity of large-scale real-time problems. Multiple comparisons and analyses have been proposed to assess the performance of the model and problem-solving capabilities. The results indicate that the proposed approach can be applied and implemented to develop a real-world humanitarian logistics network.

Keywords: Critical logistics, Primary and secondary crises, Fuzzy robust integrated programming, Meta-heuristic algorithm.

1 | Introduction

Natural disasters strike the world at various times of the year, killing countless people and causing massive financial losses to governments [2]. Floods, earthquakes, tsunamis, hurricanes, tornadoes, meteorite strikes, etc., are among natural disasters.

Any of these natural disasters can have certain adverse effects depending on their magnitude and Location. Accordingly, it is crucial to prepare for, plan for, predict, and take preventative measures in the event of a natural disaster [3]. Catastrophes, as previously stated, cause significant loss of life and property in societies and nations. Thus, getting proper emergency logistics planning in place to cope with natural disasters and crisis management is critical. Such disasters are unexpected and unpredictable in nature as science and technology advance, necessitating the availability of preventative plans as well as post-disaster emergency responses. Therefore, the minimal resources

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available should be distributed among victims in the most efficient manner possible to address their most pressing needs [4]. Natural disasters, such as floods and earthquakes, have struck many countries, including our country, with varying degrees of severity, resulting in significant financial and human losses. Clearly, planning for a post-disaster emergency response is essential, as it further improves the efficiency and effectiveness of rescue operations. A major focus of relief responses and reactions would be to minimize the loss of life and property by having preparations and plans in place to cope with the implications of disasters, as well as raising public awareness. When devising these plans, one thing to keep in mind is that the nature of natural disasters like earthquakes necessitates a swift and efficient response.

To put it another way, in such a complicated and emergency situation, the decision-maker must conduct rescue operations and resolve the injured's situation quickly and efficiently. To achieve this important goal and take timely action, it appears that having access to an effective and systematic pre-defined program with all of the required activities, sequences, and communications is required [5]. Thus, logistical preparation for the transportation of critical items needed in affected areas is one of these tasks that are also very significant. Aid provision to disaster-stricken regions, the importance of the distribution of relief goods, and the evacuation of the injured is essential and strategic, as increased efficiency of transportation of goods and casualties has a great effect on the number of survivors after the disaster [6]. Consequently, the majority of disaster relief problems have focused on optimizing routing decisions and locating ground vehicles with a variety of modeling approaches. Helicopters are used for contingency purposes due to their widespread use in medical emergencies and disaster relief. In this research, the importance of disaster relief, taking humanitarian medical aid, vaccinations, and other auxiliary products, such as tents, blankets, medicines from located warehouses and distribute them to affected areas following the sudden occurrence of natural disasters and for evacuation operations, the relocation of the wounded from affected areas and transferring them to hospitals, are taken into account sites. The aim is to find a series of routes that start and end at hospitals and take the shortest flight time possible. Another goal is for helicopters to use as little fuel as possible. Humanitarian supply chain management should be able to respond to a crisis in the shortest amount of time possible. The "last-mile delivery problem," which is becoming more prevalent in disaster relief, is also addressed in this study, in addition to the last step of the relief supply chain. Thus, a mathematical model based on last-mile distribution concepts will be considered in this research. This research aims to establish a new model for rapid response to natural disasters in the process of rapid relief to [affected] regions. The development of such measures focused on the phase of rapid response to disasters while considering real limitations is one of the fuzzy robust integrated model goals in this study. In this research, the collection and distribution of medical aids, vaccines and other auxiliary items such as tents, blankets, medicine and, … From the warehouses we have located to the damaged places after the sudden occurrence of natural disasters, and for evacuation operations, we consider the removal of the injured from the damaged areas and their transfer to hospitals. The goal is to determine the set of routes that have the shortest flight time, which should start from hospitals and end in hospitals, and also the other goal is to minimize the amount of fuel consumption by helicopters.

Efficient humanitarian supply management must be able to respond to the situation as quickly as possible and in the shortest possible time frame.

2 | Literature Review

Since critical logistics are so crucial in relief efforts, this section examines international research in crisis logistics. The following is a study of research based on the interpretation of studies published in recent years in journals. According to Rodriguez [7], to alleviate distress, resource provisioning for disaster victims must be established, as well as proper planning for these activities. To cope with crises, crisis management brings together several organizations and shares resources. As a consequence, successful operations are highly reliant on cross-organizational collaboration. Chapman [8] argued that it's vital to ensure that post-disaster relief operations are well-organized and effective and the affected population's basic needs are met. However, uncertainty also affects all facets of rescue operations in the aftermath of a disaster. The location of relief distribution centers, as well as public awareness of these locations, are critical for the pace and efficiency of relief operations. Yu [9] often used the cost of deprivation as a primary economic measure of human misery in the context of emergency supplies (logistics). An enhanced method for the effective and fair allocation of critical resources in emergency supplies has been proposed, which incorporates this economic agency to account for human suffering.

To explain the disaster response process, a dynamic planning model for a retransmission problem of extracted multi-period resource allocation is presented, with specific attention to human distress caused by delayed delivery. Vahdani [10] proposed a multi-objective two-stage integer mathematical model in which the establishment of distribution centers and warehouses with varying capacities, as well as decisions about products stored in warehouses and distribution centers built in the first step, were taken into account. Due to emergency restrictions, operational preparation was undertaken in the second step to route and distribute merchandise in affected areas, increasing overall expense, travel time, and route reliability. Then, two metacognitive algorithms, NSGA-II and MOPSO, were used to check the accuracy of the mathematical model and the performance of the proposed algorithms by numerical samples. The results of the algorithms are presented for 35 different problems. Mohammadi et al. [11] developed a two-level model for the location of transport points and distribution centers of relief goods in earthquake situations. The first level involves locating relief facilities and transport points, and the second level includes routing for transporting the injured and bodies to certain pre-determined points. In addition, three scenarios (Masha fault, Rey fault, and North fault) with probabilities of 0.35, 0.30, and 0.35 are considered based on the conditions of Tehran faults. Finally, the Epsilon constraint method and GAMZ software were used to solve this model as it was multi-objective in nature. According to the findings, ten points should be chosen for transport point establishment along highways. Zahiri et al. [12] created a multi-level model under uncertainty for planning relief goods distribution centers. Uncertainty is taken into account in parameters like demand and facility capacity, treated as triangular fuzzy.

The inventory volume of each warehouse, the number of products flow from the retailer to each warehouse and from there to the affected area are among the research variables. This research yielded the following conclusions: 1) supplier/warehouse capacity is inversely correlated with the overall cost, and growing warehouse and supplier capacity reduces costs in this model, and 2) penalty cost for unsatisfied demand plays an important role in system efficiency. By raising the penalties, all requirement points can be covered. Salehi et al. [13] proposed a probabilistic multi-period model for designing a blood distribution network in the aftermath of an earthquake. A number of blood derivatives, such as plasma and possible platelets, are also in demand. In this research, a three-level blood supply chain is considered, including: 1) donors, 2) blood collection centers, and 3) blood transfusion base. The proposed two-level model is proposed for the city of Tehran, before and after the earthquake. The number of temporary blood collection facilities is calculated at the first level, and post-earthquake scenarios, including the distribution of blood products, are run at the second level. Finally, the performance of the model is validated by Monte Carlo simulation. Khatami et al. [14] proposed a probabilistic two-level model for crisis management before and after an earthquake. The first level of this model involves locating relief stations, and the second level involves allocating these stations to crisis-stricken regions. The number of goods stored and shortage volume in each center are among the decision variables of this research. A potential earthquake in Tehran was used to assess the accuracy of the proposed multi-commodity, multi-period model. In their research, Zokaee et al. [15] introduced a three-level (tier) supply chain that included suppliers, relief distribution centers, and affected areas. This study aimed to improve victim satisfaction while also lowering prices; for this purpose, certain penalties are considered in the event of a shortage of products. Robust optimization is used to solve the proposed model, which has uncertainties in demand and cost parameters. The Alborz area, which is prone to earthquakes and other natural disasters, was used as a case study in this research. Douglas et al. [16] proposed a two-level inventory (warehouse) distribution model in the disaster relief supply chain. One of their innovations has been location risk. The transportation vehicles used in this research are heterogeneous and have varying capacities. To demonstrate the efficacy of the model, they performed a case study in Brazil. They used an advanced algorithm to solve the case study. Cavdur et al. [17] developed a two-level model for distributing relief goods in earthquake-prone areas to those affected. This research aims to reduce the distance traveled as well as to minimize the amount of unmet demand. Some scenarios based on the time of the disaster and environmental situations, such as traffic, have been defined to achieve this aim. Consideration of supply and demand equilibrium, service level, and productivity of manufacturing operation are among the innovations of this research. Finally, the proposed earthquake model is based on a case study in Turkey. Xu et al. [18] used statistical modeling and electronic systems to locate earthquake shelters after acquiring geographic information.

The proposed model is of the P-middle type, and it aims to maximize coverage while minimizing the distance to the shelter. The proposed algorithm involves three steps: 1) selection of candidate shelters, 2) analysis of the coverage range of each shelter, and 3) selection of the final shelter site. The results obtained from the implementation of the model in Yangzhou indicated the accuracy and precision of the proposed model. Ouyed and Allili [19] in her research considers a six-level chain including blood donors, blood collection centers, laboratories, blood centers, hospitals and accident centers. In order to investigate the uncertainty in the model parameters, the possibility planning method has been used. The results of numerical analysis indicate the good performance of the possibility method compared to the definitive method and the possibility of 0.9 has the best performance compared to other values. Ling et al. [20] introduced the mathematical model of the medical equipment supply chain for the prevention and control of the COVID-19 epidemic, which aimed to maximize the overall satisfaction of medical equipment and minimize the total cost of planning.

Madani et al. [21] presents a multi-stage, multi-objective back-and-forth relief network that considers the location of hospitals, local warehouses, and hybrid centers that the hospital warehouse center is in the pre-disaster stage. In the post-disaster phase, the routing of relief goods in the forward path is considered. On the way back, there are some vehicles that can transport the injured after delivery. Combined transportation facilities will transport the injured to hospitals and combined centers. Depending on the degree of difficulty, a Non-dominated Sorting Genetic Algorithm (NSGA-II) with Simulated Algorithm (SA) and Variable Neighborhood Search (VNS) is proposed to solve the proposed problems. Hallak and Miç [22], in a case study conducted north of Aleppo in Syria to locations of the relief warehouses. At first, human and economic criteria were selected by three experts and then the weight of the criteria was determined by Fuzzy Analytic Hierarchical Process (F-AHP). Finally, warehouses were evaluated and ranked by MULTIMOORA technique as Multi-Criteria Decision Making (MCDM) method. Abazari et al. [23], in their research, in the pre-disaster stage, they determined the location and number of relief centers with a specific inventory level, and then after the disaster based on the distribution program, the amount of Relief Items (RI) that should be transferred to the Demand Points (DP) and the number of the required equipment is determined. Objective functions minimize the total distance traveled by RI, the total cost, the maximum transport time between relief centers and DP, and the number of perished items. Momeni et al. [24], reducing response time with high reliability has been introduced as the main goal of their research. In this research, after the disaster, the latest information on the condition of the roads is collected by drones and motorcycles then this information analyzed by disaster management to determine the probability of each scenario. By evaluating and analyzing the collected data, route repair teams are sent to increase the reliability of the route and in the final stage, they allocate RI to the DP.

Sarma et al. [25], in their research, they have introduced a three-step model in which the demand is done with the highest priority by local agencies in the first step and other remaining demands by national and international agencies in the second step along with the restoration of local agencies in the third step is done. To illustrate the performance of their proposed model, they have provided a numerical example that has tested its convergence using LINGO software and CPLEX optimization solvers. In Dachyar and Nilsari [26], the goal of his research design improvement in disaster relief distribution information system by utilizing the Internet of Things (IoT). The method used in this research is the development of a structure system with Entity-Relationship Diagram (ERD) and Data Flow Diagram (DFD). This study reduces the cycle time of the disaster relief distribution process at the rate of 59.4% and proposes an information system design that is expected to improve accountability in the disaster relief distribution system. In Cao et al. [27], they presented a fuzzy two-level optimization model using Wenchuan earthquake data. This study presents the problem as a fuzzy tri-objective bi-level integer programming model to minimize the unmet demand rate, potential environmental risks, emergency costs on the upper level of decision hierarchy and maximize survivors' perceived satisfaction on the lower level of decision hierarchy. *Table 1* summarizes the study assessment based on the evaluation of international research on critical logistics.

Table 1. A review of international literature.

Row	Author	Single-Objective	Multi-Objective	Definitie	Robust	Probable	Fuzzy	Ordinary Logistics	Relief Logistics	Accurate Solution	Meta-Heuristic Solution
1	Abounacer et al. [28]		*	*					*	*	
2	Aghajani and Torabi [29]		*						*	*	
3	Binbin and Songchen [30]		*						*	*	
4	Boonmee et al. [31]		*						*	*	*
5	Boltürk et al. [32]		*				*		*	*	
6	Barbarosoğlu et al. [33]			*					*	*	
7	Cao et al. [2]		*						*		*
8	Chapman and Mitchell [8]		*	*					*	*	
9	Cavdur et al. [17]		*			*			*		*
10	Chen and Yu [34]		*	*					*	*	
11	Chu and Yan Zhong [35]		*			*			*		*
12	De Angelis et al. [36]		*						*		*
13	Alem et al. [16]		*						*		*
14	Fahimnia et al. [37]		*			*			*		*
15	Fereiduni and Shahanaghi [38]		*		*				*		*
16	Jha et al. [5]		*						*		*
17	Madani et al. [21]		*						*		*
18	Khatami et al. [14]		*	*			*		*		
19	Ming Zhao and Xiang Liu [39]		*	*		*					
20	Papi et al. [40]		*	*		*				*	
21	Roh et al. [41]		*		*	*		*			
22	Safaei et al. [3]		*		*	*				*	
23	Samani et al. [4]		*	*		*			*		
24	Salehi et al. [13]		*	*		*			*	*	
25	Vahdani et al. [10]		*	*		*				*	
26	Yu et al. [9]		*	*		*					
27	Zahiri et al. [12]		*		*	*		*			
28	Zokaee et al. [15]		*		*	*				*	
29	Ouyed and Allili [19]		*	*		*				*	
30	Ling et al. [20]		*	*		*				*	
31	Madani et al. [21]		*	*		*					*
32	Hallak and Miç [22]		*		*	*		*			
33	Abazari et al. [32]		*	*		*			*		
34	Momeni et al. [24]		*	*		*			*		
35	Sarma et al. [25]		*	*		*					*
36	Dachyar and Nilsari [26]	*			*	*					*
37	Cao et al. [27]		*	*		*		*			
38	Current study		*	*		*		*		*	

The application of relief logistics in crises induced by natural disasters will be addressed in this study, based on the above-mentioned national and international studies. The need to make swift decisions and carry out operations with minimal resources has contributed to developing a type of knowledge known as crisis management. In this regard, research undertaken on recent crises in various parts of the world indicates that the need to investigate and model successive developments is greater than ever. As previously mentioned, such incidents neutrally occur in a cascading fashion as a chain of interrelated disasters. They are referred to as primary and secondary crises in the literature on crisis management because the occurrence of a crisis in one region triggers a secondary crisis in the regions affected by the primary crisis.

A secondary disaster, such as a flood crisis, is possible after a primary earthquake crisis that destroys infrastructure, including water supply facilities. Thus, steps must be taken to locate, route, and allocate relief distribution centers, relief bases, temporary shelters, and optimal routes to reach the affected areas.

Given the nature of relief in major consecutive events and the importance of relief pace during the relief management process in the current model, an effort has been made to minimize relief time as much as possible in both primary and secondary crisis phases. This is achieved in the primary crisis by locating facilities to deliver RI to affected areas and routing to transport wounded people to medical centers. It is assumed that a secondary crisis occurs in the same area as the primary crisis, and it is impossible to relocate for the deployment of equipment to respond to the second incident. Following the secondary crisis outbreak, we would concentrate on minimizing relief time and transferring people whose homes have been demolished to shelters, optimizing routing for the relocation of homeless people, and maximizing relief coverage, to carry out more equitable operations. As discussed in the research literature, most studies have discussed pre-disaster components or disaster occurrences in the present case, and yet no research has been conducted on secondary disaster. Therefore, in this research, a new mathematical model in the field of secondary disaster analysis is presented, which is much more effective in terms of the volume of destruction than the primary disaster.

3 | Problem Description and Formulation

Following natural disasters, the most significant factor in determining the effectiveness of relief measures is the pace at which relief facilities and goods are made available in the affected region. According to the above, the rate of transfer of the injured are transported to treatment facilities and the homeless to temporary shelters may also be indicative of the speed at which relief is provided. Another aspect that causes more and more injured people to be satisfied is the equitable distribution of relief in the affected areas. This is especially critical when disasters like floods strike, which typically impact a wide geographic area and necessitate balancing relief services across all affected areas.

We consider a four-level supply chain structure when shipping RI to affected areas (*Fig. 1*). The key warehouses for relief supplies can be found on the first level. These warehouses are permanent or temporary facilities whose number and location are identified prior to the disaster. Relief distribution centers, shelters, and temporary medical centers are located on the second level. The number and potential location of these facilities are pre-determined. The best are chosen to be activated from among these locations to finally minimize the time it takes to transport relief goods and wounded people to medical centers. Disaster-stricken areas make up the third level. The precise number of casualties is unknown immediately after a disaster. Hence, demand for relief goods, as well as subsequent voluntary public contributions and support, are considered uncertainty parameters. Donations from the general public (voluntary public contributions) are sent to distribution centers. Warehouses contain a basic inventory of different relief goods, and government assistance is sent to warehouses at the start of the relief period.

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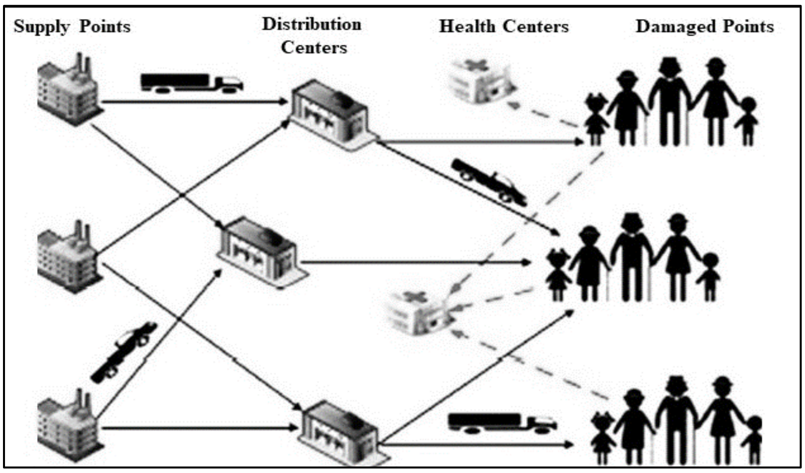

Fig. 1. Relief network investigated in this research.

3.1 | Assumptions

The assumptions of the model are as follows:

I. The number and location of key warehouses are well-defined.
II. The number and candidate location for temporary distribution centers (midpoints), medical centers, and shelters are well-defined.
III. The number and location of affected areas are well-known after the disaster.
IV. Supply points or warehouses have a certain capacity for receiving and sending goods.
V. Midpoints, or temporary distribution centers, shelters, and temporary medical centers, have a certain capacity to receive and send supplies and casualties.
VI. Network arcs are linking ways from supply points to distribution centers, from distribution centers to affected points, from affected points to shelters and medical centers, and from supply points to affected points.
VII. Various types of relief goods are considered.
VIII. A multi-period mathematical model is considered.
IX. The products' volume and weight are well-defined.
X. Each vehicle has a certain transportation capacity.
XI. Each mode of transportation has its own set of routes to follow.
XII. Roads can be blocked after a disaster.
XIII. Affected point demand is taken into account as a reliable uncertainty parameter.
XIV. Since primary and secondary crises are possible, the response phases to primary and secondary crises will be scheduled separately.
XV. A secondary crisis has a certain probability of occurring, but it may not occur in the same region even after the primary crisis.
XVI. A two-stage crisis occurs, with the second stage occurring after the first.
XVII. People who have been injured are separated into two categories: those who need medical attention and those who need shelter.

3.2 | Notations

Indexes

I: the warehouse node.

J: the temporary procurement center node.

K: the node of the damaged place in the primary and secondary disasters.

M: the set of candidate nodes of the temporary medical center.

N: the shelter node.

L: the type of vehicles (i.e., trucks and helicopters).

C: the relief commodity.

D: the type of injury (i.e., injuries classification based on treatment or their transfer to shelters).

S: the scenario in the primary and secondary disasters–two scenarios are considered in the mathematical model, which are the first scenario (an earthquake as the primary disaster and flood as the secondary disaster), and the second scenario (an earthquake as the primary disaster and fire as the secondary disaster).

Parameters

P_s: the probability of scenario s in the primary and secondary crises.

$T\delta_{lij}$: the transfer-shipment time of vehicle L between nodes i and j.

$\delta\prime_{ljj\prime}$: the transfer-shipment time of vehicle L between nodes j and j'.

Tc_{ljk} : the transfer-shipment time of vehicle L between nodes j and k.

$Tcc_{lkk\prime}$: the transfer-shipment time of vehicle L between nodes k and k'.

U_{knd}: the transfer time of injured individuals of type d from demand node k to shelter n in the secondary disaster.

z_{kmd}: the transfer time of injured individuals of type d from demand node k to medical center m in the primary disaster.

B_0: the relief time in the primary disaster.

B_1: the relief time in the secondary disaster.

dem_{kcs}: the demand for commodity c in node k under scenario s.

$capp_{ic}$: the amount of commodity c that can be supplied by node i.

o_{kd}: the number of injured individuals of type d in node k in the primary and secondary disasters.

ca_{nd}: the capacity of shelter n for receiving injured individuals of type d in the secondary disaster.

se_{md}: the capacity of medical center m for receiving injured individuals of type d in the primary disaster.

$ccap_j$: the volumetric capacity of temporary procurement center j.

β_{kd}: the percentage of injured individuals type d in disaster center k.

MM: is a large number.

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Decision parameters

Q_{lijcs}: the amount of commodity c supplied from I and stored in node I, by vehicle l under scenario s.

$Q^1_{lijj\prime cs}$: the amount of commodity c sent by temporary procurement center j to procurement center j' through vehicle L under scenario s.

Y_{ljkcs}: the amount of commodity c transported by vehicle L from node j to node k under scenario s.

$Y^1_{ljk\prime kcs}$: the amount of commodity c transported by vehicle L from center k' to center k under scenario s.

T_{LKs}: the arrival time of vehicle L in disaster center k under scenario s.

X_{kcs}: the stored amount of commodity c in node k under scenario s.

AA_{lkmds}: the number of injured individuals type d transferred by vehicle L from node k to medical center m under scenario s.

A_{lknds}: the number of injured individuals type d transferred by vehicle L from node k to shelter n under scenario s.

b_{kcs}: the shortage of commodity c in node k under scenario s.

e_{kds}: the number of the unhandled injuries type d in node k under scenario s.

ZZ_{js}: is 1 if a temporary procurement center is established in node j under scenario s, otherwise, it is zero.

H_{ms}: is 1 if a medical center is established in node m under scenario s, otherwise, it is zero.

D_i^+: is the positive deviation from the goal considered by the objective function i.

D_i^-: is the negative deviation from the goal considered by the objective function i.

Hs_{ns}: is 1 if shelter n is established under scenario s, otherwise, it is zero.

X^1_{lijs}: is 1 if vehicle L moves from supplier i to temporary procurement center j under scenario s, otherwise it is zero.

$X^2_{lijj\prime s}$: is 1 if vehicle L moves from temporary procurement center j to temporary procurement center j' under scenario s, otherwise it is zero.

X^3_{ljks}: is 1 if vehicle L moves from temporary procurement center j to disaster center k under scenario s, otherwise it is zero.

$X^4_{ljkk\prime s}$: is 1 if vehicle L moves from disaster center k to disaster center k'under scenario s; otherwise it is zero.

X^5_{lkms}: is 1 if vehicle L moves from disaster center k to medical center m under scenario s, otherwise it is zero.

X^6_{lkns}: is 1 if vehicle L moves from disaster center k to shelter n under scenario s, otherwise it is zero.

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3.3 | Mathematical Modeling

$$MIN\ Z1C =$$

$$p_s\left(\sum_i\sum_j\sum_k\sum_l\sum_m\sum_n T_{lij} * X^1_{lijs} + '_{ljj'} * X^2_{lijjs'} + Tc_{ljk} * X^3_{ljks} + Tcc_{lkk'} * X^4_{ljkk's} + U_{knd} * X^6_{lkns} + z_{kmd} * X^5_{lkms}\right) \tag{1}$$

$$MIN\ Z2C =$$

$$p_{kc} + q_{kcs} \geq dem_{kcs} \quad \text{for all } k, c. \tag{2}$$

The first objective function is to minimize vehicle routing scheduling (time) for the phase of responding to the primary crisis and the location of temporary relief logistics centers during the primary crisis, while the second objective function is to minimize the time it takes to move wounded people to shelter during the secondary crisis.

$$MIN\,C = p_s\left(\sum_L\sum_K T_{LKs}\right). \tag{3}$$

The third objective function is to minimize the golden relief time and movement of the wounded to a temporary medical center in the secondary crisis.

$$MAX\,C = p_s\left(\sum_i\sum_j\sum_k\sum_l\sum_m\sum_n X^1_{lijs} + X^2_{lijj's} + X^3_{ljks} + X^4_{ljkk's} + X^6_{lkns} + X^5_{lkms}\right) + D^-_4 + D^+_4. \tag{4}$$

The fourth objective function is to maximize the coverage of disaster-stricken areas by routing vehicles, transferring relief goods, and transferring wounded people in secondary crises.

Table 2. The objective function identifier of the problem is as follows.

	Objective Function	Method
Primary crisis	Minimizing the amount of time it takes to transport relief supplies and transfer injured people to medical centers	Location of the temporary procurement (logistics) center Location of the shelter Location of the medical center Optimal routing between the warehouse and the temporary procurement center Routing between temporary procurement centers (transfer shipment) Routing between procurement center and crisis centers
Secondary crisis	Minimizing the time it takes for injured people to be transferred to a shelter. Maximizing the coverage of crisis-stricken areas	Routing the transportation of the injured people to the shelter Allocation of temporary warehouses to crisis centers and shelter centers to crisis-stricken areas

3.4 | Fuzzy Programming

$$\sum_l\sum_j Q_{lijcs} + \sum_l\sum_{j'\neq j}\sum_J Q^1_{lij'cs} \leq capp_{ic} \qquad \text{for all } i, c, s. \tag{5}$$

In *Constraint (5)*, the volume of merchandise passing between the warehouse and the temporary procurement centers should be less than the transportation vehicles' capability.

$$\sum_l\sum_i\sum_c Q_{lijcs} + \sum_l\sum_{j'\neq j}\sum_c\sum_i Q^1_{lij'jcs} \leq ccap_j \qquad \text{for all } j, s. \tag{6}$$

In *Constraint (6)*, the volume of merchandise submitted to temporary procurement centers should be smaller than the available storage space to temporary distribution centers.

$$\sum_{l}\sum_{i}Q_{lijcs}+\sum_{l}\sum_{j'\neq j}\sum_{i}Q^{1}{}_{lij'jcs}=\sum_{l}\sum_{k}Y_{ljkcs}+\sum_{l}\sum_{k'\neq k}\sum_{K}Y^{1}{}_{lJkk'cs} \qquad \text{for all } j,c,s. \tag{7}$$

In *Constraint (7)*, according to this constraint, the volume of goods arriving at each temporary procurement center from a warehouse or other temporary procurement centers should be directed to the crisis center, or excess goods may be sent to other temporary procurement centers.

$$\sum_{l}\sum_{i}Q_{lijcs}+\sum_{l}\sum_{j'\neq j}\sum_{i}Q^{1}{}_{lij'jcs}=\sum_{l}\sum_{k}Y_{ljkcs}+\sum_{l}\sum_{k'\neq k}\sum_{K}Y^{1}{}_{lJkk'cs} \qquad \text{for all } j,c,s. \tag{8}$$

Demands of crisis centers are received either directly from supply centers (the sender), or they must be satisfied by equipment sent to another crisis center. On the other side, crisis centers may store inventory or require a deficit.

$$\sum_{l}\sum_{m}AA_{lkmds}+e_{kd}=o_{kd} \qquad \text{for all } k,d,s. \tag{9}$$

Constraint (9) transfer of the injured people from every region struck by a primary crisis to medical centers.

$$\sum_{l}\sum_{n}A_{lknds}+e_{kd}=o_{kd} \qquad \text{for all } k,d,s. \tag{10}$$

In *Constraint (10)*, the injured people from any region struck by a secondary crisis can be taken to shelter centers by equipment that delivers goods.

$$\beta_{kd}\sum_{l}\sum_{m}AA_{lkmds}+\left(1-\beta_{kd}\right)*\sum_{l}\sum_{n}A_{lknds}\leq o_{kd} \qquad \text{for all } k,d,s. \tag{11}$$

In *Constraint (11)*, this constraint specifies the number of wounded persons.

$$\sum_{l}\sum_{k}AA_{lkmds}\leq se_{md} \qquad \text{for all } m,d,s\ . \tag{12}$$

In *Constraint (12)*, the number of injured people transferred to medical centers must be smaller than the admission capacity of the medical center.

$$\sum_{l}\sum_{k}A_{lknds}\leq ca_{nd} \qquad \text{for all } n,d,s. \tag{13}$$

In *Constraint (13)*, the number of injured persons who have become homeless due to a secondary crisis and need to be moved to shelter centers must be smaller than the admission capacity of the shelter.

$$\sum_{i}\sum_{j}\sum_{k}\sum_{l}\sum_{m}\sum_{n}T_{lij}*X^{1}_{lijs}+'_{ljj'}*X^{2}_{lijj's}+Tc_{ljk}*X^{3}_{ljks}+Tcc_{lkk'}*X^{4}_{ljkk's}+z_{kmd}*X^{5}_{lkms}\leq B_{0} \tag{14}$$

$$\text{for all } s.$$

According to *Constraint (14)*, the pre-secondary crisis timetable for shipping relief supplies transversely must be shorter than the permitted time.

$$\sum_{i}\sum_{j}\sum_{k}\sum_{l}\sum_{m}\sum_{n}T_{lij}*X^{1}_{lijs}+'_{ljj'}*X^{2}_{lijj's}+Tc_{ljk}*X^{3}_{ljks}+Tcc_{lkk'}*X^{4}_{ljkk's}+U_{knd}*X^{6}_{lkns}\leq B_{1}. \tag{15}$$

Constraint (15) specifies that both transport and displacement times in secondary crisis management should be shorter than the permitted time.

$$\sum_{c} Q_{lijcs} \leq MM * X^{1}_{lijs} \quad \text{for all } l,i,j,s. \tag{16}$$

In *Constraint (16)*, there is limited communication between routing and the volume of merchandise shipped from the supplier to the temporary procurement site.

$$\sum_{c} Q_{lijcs} \leq MM * X^{1}_{lijs} \quad \text{for all } l,i,j,s. \tag{17}$$

In *Constraints (17)*, there is limited communication between routing and the volume of merchandise shipped from the supplier and temporary procurement to the temporary procurement site.

$$\sum_{c} Y_{ljkcs} \leq MM * X^{3}_{ljks} \quad \text{for all } l,j,k,s. \tag{18}$$

In *Constraints (18)*, there is limited communication between routing and the volume of merchandise shipped from the temporary procurement center to disaster-stricken areas.

$$\sum_{c} Y^{1}{}_{ljkk'cs} \leq MM * X^{4}_{ljkk's} \quad \text{for all } l,j,k,k' \neq k,s. \tag{19}$$

In *Constraints (19)*, there is limited communication between routing and the volume of merchandise shipped from the temporary procurement center and disaster-stricken areas to disaster-stricken areas.

$$\sum_{j} X^{1}_{lijs} \leq 1 \quad \text{for all } l,i,s. \tag{20}$$

Constraint (20) specifies the limited number of times the equipment will leave the supplier. Based on this constraint, the equipment can only depart the supplier once and move toward the temporary procurement center.

$$\begin{aligned} &\sum_{L} X^{1}_{lijs} + \sum_{j' \neq j} \sum_{L} X^{2}_{lj'js} \leq 1 \quad \text{for all } i,j,s, \\ &\sum_{j' \neq j} \sum_{L} X^{2}_{lijj's} \leq 1 \quad \text{for all } i,j,s. \end{aligned} \tag{21}$$

Constraint (21) limited number of times equipment arrives at the temporary procurement center.

$$\sum_{i} \sum_{l} X^{1}_{lijs} + \sum_{i} \sum_{j' \neq j} \sum_{l} X^{2}_{lij'js} = \sum_{k} \sum_{l} X^{3}_{ljks} + \sum_{i} \sum_{j' \neq j} \sum_{l} X^{2}_{lijj's} \quad \text{for all } j,s. \tag{22}$$

Constraint (22) states that the number of arrivals to temporary procurement centers should be equal to the number of departures from these centers, as they do not store transportation vehicles.

$$\sum_{k} X^{3}_{ljks} \leq 1 \quad \text{for all } l,j,s. \tag{23}$$

Constraint (23) limited number of times equipment is brought into the crisis center.

$$\sum_{i} \sum_{l} X^{1}_{lijs} + \sum_{i} \sum_{j' \neq j} \sum_{l} X^{2}_{lij'js} = ZZ_{j} \quad \text{for all } j,s. \tag{24}$$

Constraint (24) constrained construction of temporary procurement centers.

$$\sum_{l} \sum_{k} \sum_{d} AA_{lkmds} \leq MM * H_{ms} \quad \text{for all } m,s. \tag{25}$$

Constraint (25) constrained construction of temporary medical centers.

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$$\sum_{i} X^{1}_{lijs} \geq \sum_{i}\sum_{j'\neq j} X^{2}_{lijj's} \quad \text{for all } j,l,s. \tag{26}$$

Constraint (26) requiring early routing between suppliers and temporary procurement centers

$$\sum_{j} X^{3}_{ljks} \geq \sum_{j}\sum_{k'\neq k} X^{4}_{ljkk's} \quad \text{for all } l,K,s\ . \tag{27}$$

Constraint (27) requiring early routing between temporary distribution centers and disaster-stricken areas.

$$\sum_{d} AA_{lkmds} \leq MM * X^{5}_{lkms} \quad \text{for all } l,k,m,s. \tag{28}$$

Constraint (28) routing between the disaster-stricken area and the medical center.

$$\begin{aligned} &\sum_{d} A_{lknds} \leq MM * X^{6}_{lkns} \quad \text{for all } l,k,n,s, \\ &\sum_{l}\sum_{k}\sum_{d} A_{lknds} \leq MM * Hs_{ns} \quad \text{for all } n,s. \end{aligned} \tag{29}$$

Constraint (29) routing between the crisis-stricken area and the shelter, as well as the erection of the shelter center.

$$\begin{aligned} &T_{LK} \leq \sum_{I}\sum_{J}\sum_{C} T_{lijc} * X^{1}_{lijs} + \sum_{J}\sum_{C} Tc_{ljkc} * X^{3}_{ljks} + \sum_{K'\neq K}\sum_{J}\sum_{C} Tcc_{lk'Kc} * X^{4}_{ljk'ks}. \\ &T_{LK} \leq 48 \end{aligned} \tag{30}$$

In *Constraint (30)*, the equipment transportation time for distributing goods among crisis area and the golden time for relief should be less than 48 hours.

According to Inuiguchi and Ramık [42], the above model can be rewritten as

$$\begin{aligned} &\text{Min } Z = f.y + \left(\frac{c_{(1)} + c_{(2)} + c_{(3)} + c_{(4)}}{4}\right).x, \\ &A.x \geq \left(1-\alpha_{m}\right).d_{(1)} + \alpha_{m}.d_{(2)} \ \text{ for all } m, \\ &B.x = 0, \\ &s.x \leq N.y, \\ &0.5 \leq \alpha_{m} \leq 1, \\ &x \geq 0 \text{ for all } m, \\ &y \in \{0,1\}. \end{aligned} \tag{31}$$

As previously mentioned, the model proposed in the previous section is a fuzzy linear model. This section will use robust programming and Bertsimas-Sim approach to add demand uncertainty to the model.

Thus, *Constraint (8)* will be modified to the Bertsimas model. Hence, the proposed model will be linear. According to the findings of this research, customer demand is one of the significant parameters whose values can surpass nominal values. Therefore, taking this parameter into account in uncertain situations will help the proposed model get closer to the problem reality. Customer demand uncertainty will be addressed using robust programming and the Bertsimas-Sim approach.

The robust optimization approach looks for solutions that are either optimal or near-optimal and are likely to be justified. One of four approaches to considering uncertainty in robust programming is the Bertsimas-Sim approach. We will briefly discuss this method in this section. The following linear programming model is considered for this purpose:

$$\begin{aligned}&\text{Min } Z\sum_{j} C_j X_j,\\&\text{s.t,}\\&\quad Ax \;\; b.\end{aligned} \tag{32}$$

In this model, it is assumed that only the right-hand coefficients in the constraints, i.e., matrix A, have non-crisp values, and the entries of this matrix, i.e., a_{ij} s, fluctuate in the $[\tilde{a}_{ij} - \hat{a}_{ij}, \tilde{a}_{ij} + \hat{a}_{ij}]$ range, where $\tilde{a}_{ij}$ and $\hat{a}_{ij}$ represent the nominal values and maximum deviation of parameter a_{ij}, respectively. Bertsimas and Sim's proposed robust model is in the form of *Eq. (33)*.

$$\begin{aligned}&\text{Min } Z\sum_{j} C_j X_j,\\&\text{s.t.} \sum_{j} \tilde{a}_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} \mu_{ij} \le b_i \qquad \text{for all } i,\\&z_i + \mu_{ij} \ge \hat{a}_{ij} x_{ij} \qquad \text{for all } i, j,\\&z_j, \mu_{ij} \ge 0 \qquad \text{for all } i, j.\end{aligned} \tag{33}$$

In these equations, and μ_{ij} are dual slack variables, and Γ_i, or the uncertainty budget, represent the degree of conservatism chosen based on the importance of the constraint as well as the decision-maker's risk-taking. The mathematical model presented in the demand section is taken as fuzzy by the proposed procedure. Thus, the demand-associated constraint is modified as follows:

$$\begin{aligned}&\sum_{l}\sum_{j} Y_{ljkcs} + \sum_{l}\sum_{k1 \neq k}\sum_{J} Y^{1}_{lJk'kcs} \ge (1-\alpha)\text{dem}_{kcs(1)} + \alpha.\text{dem}_{kcs(2)} + \Gamma_{kc} p_{kc} + q_{kcs} + X_{kcs} - b_{kcs} \text{ for all } k, c, s,\\&\sum_{l}\sum_{j} Y_{ljkcs} + \sum_{l}\sum_{k1 \neq k}\sum_{J} Y^{1}_{lJk'kcs} \le (1-\alpha)\text{dem}_{kcs(4)} + \alpha.\text{dem}_{kcs(3)} + \Gamma_{kc} p_{kc} + q_{kcs} + X_{kcs} - b_{kcs} \text{ for all } k, c, s,\\&p_{kc} + q_{kcs} \ge \text{dem}_{kcs} \qquad \text{for all } k, c.\end{aligned} \tag{34}$$

4 | Findings

The relief logistics and crisis management are of paramount importance since the disaster response phase in crisis management must occur in the shortest possible time. The emergence of primary crises, according to evaluations, also triggers and intensifies secondary crises. The longer the response phase of the primary crisis is postponed, the more detrimental the effect of the secondary crisis will be on the crisis-stricken region. In this problem, one of the fundamental principles under consideration when it comes to primary and secondary crises is humanitarian logistics. There are four levels to this evaluation: Warehouses for the storage of relief supplies in large quantities, temporary procurement centers to support the logistical processes of delivering relief goods to crisis-stricken areas during both primary and secondary crises, crisis-stricken areas in need of prompt and effective treatment, and medical centers responsible for treating the injured people during the primary crisis, and shelter centers responsible for treating the injured people during the secondary crisis. Since critical incidents occur in both the primary and secondary stages, two types of transportation equipment are used in this evaluation: land and airborne. The volume of portable RI, as well as the cost and timing of relocation, all influence the equipment selection. Two scenarios have been considered likely in this evaluation. A major earthquake is expected in the first scenario. The occurrence of one of the flood or fire components is considered in the second scenario or secondary incident. Transportation capacity and golden relief time are the two fundamental constraints that are considered for transportation equipment.

In addition, the formulated mathematical model is multi-objective. The first objective [function] is vehicle routing time for the crisis response phase. This evaluation seeks to minimize the time it takes to transport the relief cargo, provide relief to the wounded, and move them to temporary medical centers.

The second objective function is to minimize the golden time for relief, with a maximum of 48 h. The third objective function is to minimize routing during the secondary crisis and relocate victims to pre-determined shelters. In general, natural disasters are so complex that humans are still unable to predict the exact time of a disaster, despite the implementation of thousands of prevention techniques in the form of networks all around the world and continuous data analysis using powerful machines. This section will evaluate and analyze the results of the model.

4.1 | Numerical Examples

An example with random data was examined to validate the model's accuracy, according to the modeling. Thus, the most important parameters of the model are:

I. Time spent transporting equipment L from the ith warehouse to the jth node (in min).

Table 3. Time spent on transporting from the warehouse to the middle node.

	J1	J2	J3	J4
L1.S1	12	15	15	14
L1.S2	13	14	15	12
L1.S3	12	13	13	15
L2.S1	14	12	13	14
L2.S2	14	12	13	14
L2.S3	15	13	14	12
l3.S1	14	12	15	14
l3.S2	14	13	13	12
l3.S3	13	13	13	14
l4.S1	12	15	13	13
l4.S2	14	13	13	12
l4.S3	12	14	14	13

This parameter represents the time it takes for the shipment from the supplier to the temporary procurement center in the primary crisis scenario. Depending on the volume of demand, relief supplies are distributed to temporary procurement centers.

II. Time spent transporting equipment L from the jth node to the j'th node (in min).

Table 4. Transfer shipment time between j and X.

	J1	J2	J3	J4
L1.J1	-	8	10	10
L1.J2	8	-	11	8
L1.J3	11	9	-	10
L1.J4	13	11	11	-
L2.J1	-	10	12	12
L2.J2	13	-	9	13
L2.J3	9	11	-	10
L2.J4	11	9	11	-
l3.J1	-	9	11	13
l3.J2	9	-	13	9
l3.J3	9	10	-	13
l3.J4	10	12	9	-
l4.J1	-	9	11	8
l4.J2	12	-	11	10
l4.J3	11	10	-	10
l4.J4	11	11	10	-

Table 4 indicates the length of transportation based on primary and secondary incidents during the primary crisis. Following the secondary crisis, the same strategy is followed for a shorter time. As we all know, during the primary crisis, procurement centers' stockpiles (inventory) are larger than the needs of crisis-stricken regions. Hence, in the event of a secondary crisis, inventory is transferred between temporary

Procurement centers to reduce the response time phase to the secondary crisis. Thus, relief supplies arrive at crisis centers in a shorter period of time during the secondary crisis phase.

III. Time spent transporting equipment L from the jth node to the kth node (in min).

Table 5. Time spent transporting equipment L from the jth node to the kth node.

	K1	K2	K3
L1.J1	7	9	9
L1.J2	5	10	7
L1.J3	8	11	11
L1.J4	6	5	11
L2.J1	12	9	6
L2.J2	11	10	6
L2.J3	9	11	5
L2.J4	9	12	9
l3.J1	7	9	10
l3.J2	8	12	11
l3.J3	7	6	6
l3.J4	7	8	10
l4.J1	6	8	11
l4.J2	9	9	5
l4.J3	11	8	9
l4.J4	10	6	11

This parameter is set in the event of a primary or secondary crisis. Since the transport distance does not change in either case, the transport time remains constant. Two types of road vehicles (vans and trucks) and two types of airborne equipment (helicopter and quadrotor) are taken into account in this parameter.

IV. Time spent transporting equipment L from the kth node to the k'th node (in min).

Table 6. Transfer shipment time for k.

	K1	K2	K3
L1.K1	6	4	4
L1.K2	6	7	5
L1.K3	6	4	7
L2.K1	7	4	5
L2.K2	5	6	5
L2.K3	6	6	5
l3.K1	5	4	4
l3.K2	6	5	6
l3.K3	7	4	5
l4.K1	5	5	5
l4.K2	6	6	6
l4.K3	6	6	6

According to the definition of transfer shipment status between crisis-stricken nodes during primary and secondary crises, the status of transmission of relief goods is considered to be exchanging relief goods. *Table 6* displays the time spent transporting this parameter.

V. Time spent transporting a d-type injured person from the kth demand node to the nth shelter during a secondary crisis (in min).

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Table 7. Time spent transporting a d-type injured person from the kth demand node to the nth shelter during a secondary crisis.

	D1	D2
K1.N1	15	13
K1.N2	13	15
K2.N1	12	14
K2.N2	14	15
K3.N1	14	14
K3.N2	13	15

It is assumed that the wounded need medical attention during the primary crisis. This parameter specifies the time it takes for wounded people to be transported to temporary medical centers.

VI. Time spent transporting a d-type injured person from the kth demand node to the mth medical center during a primary crisis.

Table 8. Time spent transporting a d-type injured person from the kth demand node to the mth medical center during a primary crisis.

	D1	D2
K1.M1	14	13
K1.M2	14	15
K2.M1	13	12
K2.M2	14	12
K3.M1	15	13
K3.M2	15	15

This parameter takes into account two different types of injury. M1 injury: during the primary crisis, it is assumed that the wounded need medical attention. M2 injury: during a secondary crisis, it is assumed that people need shelter and are moved there.

VII. Demand for the product C in the Kth node under scenario s.

Table 9. Demand for the product c in the kth node under scenario s (θ = 1).

	SS1	SS2
K1.C1	32	63
K1.C2	74	42
K1.C3	44	67
K2.C1	54	79
K2.C2	39	36
K2.C3	50	66
K3.C1	68	54
K3.C2	76	57
K3.C3	66	48

Table 10 introduces the required demand in crisis-stricken areas assumed in two primary crisis scenarios.

Table 10. Demand for the product c in the kth node under scenario s (θ = 2).

	SS1	SS2
K1.C1	89	81
K1.C2	89	97
K1.C3	83	90
K2.C1	99	86
K2.C2	92	99
K2.C3	88	91
K3.C1	99	91
K3.C2	87	92
K3.C3	98	93

Table 11 introduces the required demand in crisis-stricken areas assumed in two primary crisis scenarios.

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Table 11. Demand for the product c in the kth node under scenario s (θ = 3).

	SS1	SS2
K1.C1	117	117
K1.C2	126	136
K1.C3	109	137
K2.C1	101	128
K2.C2	123	120
K2.C3	121	102
K3.C1	132	101
K3.C2	113	129
K3.C3	102	125

Table 12. Demand for the product c in the kth node under scenario s (θ = 4).

	SS1	SS2
K1.C1	148	160
K1.C2	168	168
K1.C3	173	162
K2.C1	179	161
K2.C2	167	159
K2.C3	176	171
K3.C1	176	143
K3.C2	170	156
K3.C3	144	157

Table 12 introduces the required demand in crisis-stricken areas assumed in two secondary crisis scenarios.

The collected information was divided based on two primary and secondary incident scenarios based on incidents, e.g., floods, earthquakes, fires, and hurricanes. According to the papers evaluated, 70% of the last 20 events were primary crises, and 30% of primary crises had turned into secondary crises. Therefore, the likelihood of scenarios occurring is adjusted. Following the GAMS coding, the following conclusions are initially drawn:

4.2 | Computational Experiments

We always maximize one of the objectives in solving a mathematical model accurately using the augmented epsilon-constraint method, as long as we define the maximum permissible limit for the other objectives as constraints. The mathematical representation of a bi-objective problem will be as follows:

$$\text{Min } f_1(x) \quad \text{Subject to} \quad f_2(x) \le \varepsilon_2, f_3(x) \le \varepsilon_3, \ldots, f_p(x) \le \varepsilon_p, x \in S. \tag{35}$$

The Pareto edge of the problem can be found by shifting the values of the right-hand side of the new constraints of *ε*s. One of the main drawbacks of the epsilon-constraint method is the high volume of computations needed since several different ε_i values (p-1) must be tested for each of the objective functions translated to constraint. Obtaining the maximum and minimum of each objective function without considering other objective functions in the $x \in S$ space is one of the most common approaches to implementing the epsilon-constraint method. The range associated with each objective function is then calculated using the values obtained in the previous step. If the maximum and minimum values of the objective functions are referred to as f_i^{max} and f_i^{min}, respectively, the range of each is determined as follows:

$$r_i = f_i^{max} - f_i^{min}. \tag{36}$$

The r_i range is divided into the q_i range. Then, we can get $q_i + 1$ different values for ε_i using the following formula:

$$\varepsilon_i^k = f_i^{max} - \frac{r_i}{q_i} \times k \quad k = 0,1,\ldots,q_i. \tag{37}$$

In the equation above, K represents the number of the new point associated with ε_i. The above multi-objective optimization problem can be reduced to $\prod_{i=2}^{p}(q_i+1)$ single-objective optimization sub-problems using the epsilon-constraint method. Each sub-problem has an S solution space since they will even be more constrained by inequalities related to objective functions $f_2,\ldots,f_p$. Finally, the following values are obtained for each variable:

Table 13. Parameters of the Epsilon-constraint method.

r2	72	r3	942	r4	3072
Li	4	Li	4	Li	4
NIS2	110	NIS3	258	NIS4	5928
PISF2	182	PISF3	1200	PISF4	9000
ϑ	0.0001	ϑ	0.0001	ϑ	0.0001

The values of εs are then calculated using *Eq. (37)*.

Table 14. Epsilon values.

ε4	ε3	ε2
5928	258	110
6696	493.5	128
7464	729	146
8232	964.5	164
9000	1200	182

Finally, we used GAMS software to solve the augmented epsilon-constraint model for each of the obtained εs. *Table 15* lists the Pareto optimal solutions found:

Table 15. Pareto optimal solutions in the augmented epsilon-constraint method.

ε	First Objective Function	Second Objective Function	Third Objective Function	fourth Objective Function
1	73536	110	258	5928
2	80601	128.2	494.1	6696
3	87667	146.4	729	7464
4	94848	164	964	8232
5	96546	173	1034	9431

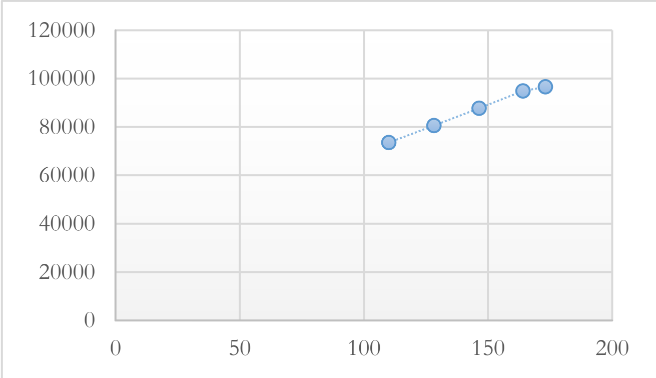

Fig. 2. Pareto boundary of optimal solutions of the first and second objective functions.

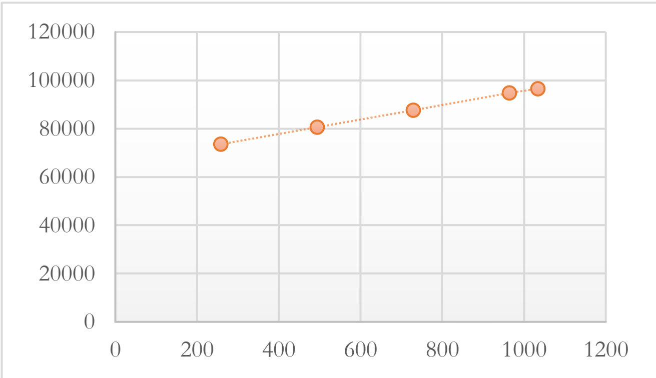

Fig. 3. Pareto boundary of optimal solutions of the first and third objective functions.

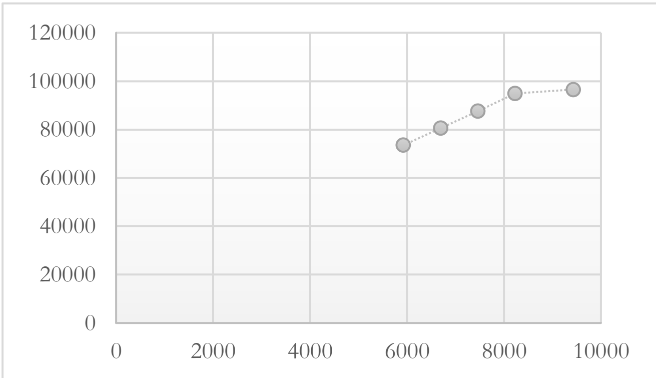

Fig. 4. Pareto boundary of optimal solutions of the first and fourth objective functions.

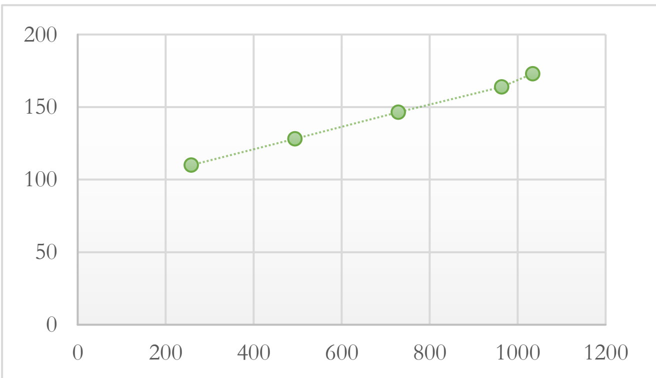

Fig. 5. Pareto boundary of optimal solutions of the second and third objective functions.

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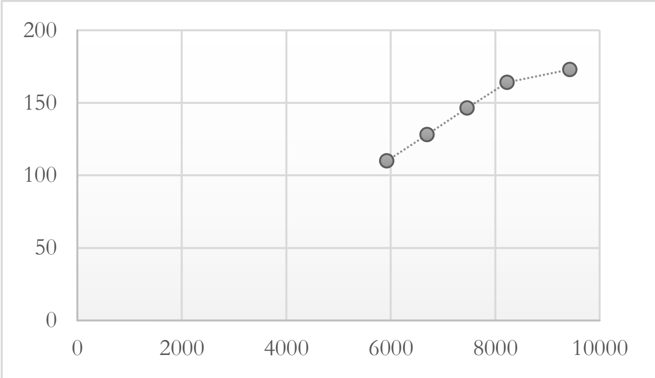

Fig. 6. Pareto boundary of optimal solutions of the second and fourth objective functions.

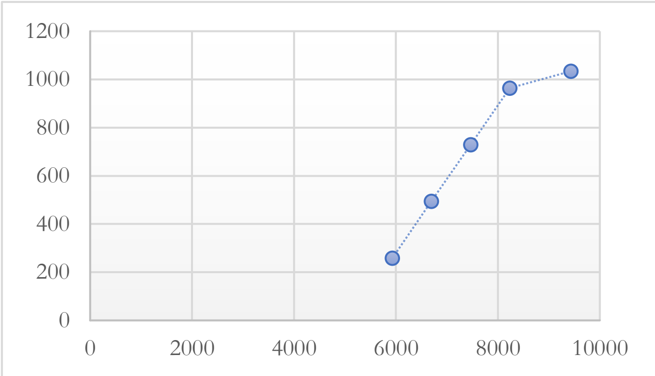

Fig. 7. Pareto boundary of optimal solutions of the third and fourth objective functions.

4.3 | Parameters Tuning

The use of performance measures is one way to address such problems. The following are some of these measures:

Number of Pareto solutions

This criterion is the number of output solutions per algorithm execution. This criterion is defined as the number of output solutions produced while running each algorithm in comparing several algorithms. Obviously, the more Pareto solutions a method has, the more desirable it is.

Average distance to the optimal solution (Mean Ideal Distance (MID))

This criterion is used to determine the average distance of Pareto solutions from the origin of coordinates. The lower the value of this criterion, the higher the efficiency of the algorithm, as seen in the following equation.

$$\mathrm{MID}=\frac{\sum_{i=1}^{n}\sqrt{\left(\frac{f1_i-f1_{best}}{f1_{total}^{max}-f1_{total}^{min}}\right)^2+\left(\frac{f2_i-f2_{best}}{f2_{total}^{max}-f2_{total}^{min}}\right)^2}}{n}. \tag{38}$$

CPU time

CPU time is a critical measure in large-scale problems. Thus, the CPU time of an algorithm is used as a criterion for evaluating its quality.

Maximum Scattering (MS)

The distance index is defined as *Eq. (35)*:

$$MS = \sqrt{\sum_{i=1}^{I}\left(\min f_i - \max f_i\right)^2}. \quad (39)$$

Scattering index of non-dominated solutions (Spread of Non-dominance Solutions (SNS))

This index is used to determine the scattering and diversity of the Pareto solutions that have been found:

$$SNS = \sqrt{\frac{\sum_{i=1}^{n}\left(MID - C_i\right)^2}{n-1}}. \quad (40)$$

$$C_i = \sqrt{f1_i^2 + f2_i^2}.$$

4.4 | Results and Comparative Study of Solution Methods

It is time to design the experiment using the Taguchi method after you have designed the problem. The Taguchi method, as previously mentioned, reduces the number of experiments needed to set the parameters. We begin by determining the parameters we want to set in each algorithm. Parameter levels and orthogonal arrays for experiments are obtained using Minitab software. We tested the algorithms at the same determined levels and repeated them ten times after deciding the number of tests for each algorithm. The results of these ten experiments were then averaged. Then, we made them unweighted, plotted S/N diagrams, and determined the better parameters. We must first obtain and note the levels of each algorithm. To do so, relevant papers were studied, and candidate levels were identified from among them, as presented in *Table 16*.

Table 16. Various levels for the parameters of each algorithm.

Algorithm	Algorithm Parameters	Parameter Level Level 1	Level 2	Level 3
NSGA-II	Pc	0.7	0.8	0.9
	Pm	0.05	0.1	0.15
	N-pop	50	100	150
	Max-iteration	2*(i+j+k+l+o)	3*(i+j+k+l+o)	4*(i+j+k+l+o)
NRGA	Pc	0.7	0.8	0.9
	Pm	0.05	0.1	0.15
	N-pop	50	100	150
	Max-iteration	2*(i+j+k+l+o)	3*(i+j+k+l+o)	4*(i+j+k+l+o)

Finally, the experiments were designed, and the L9 orthogonal arrays were chosen for the NSGA-II and NRGA algorithms using the Minitab 16 software. Response values for the Taguchi method were obtained after running the algorithms for each of the preceding experiments. *Table 17* illustrates these values and orthogonal arrays.

Table 17. L9 orthogonal array and the computational results of the NSGA-II and NRGA algorithms.

Test	Pc	Pm	N-pop	Max-Iteration	NRGA Response	NSGA-II Response
1	1	1	1	1	6.4234e-006	4.497e-006
2	1	2	2	2	9.5085e-006	9.3753e-006
3	1	3	3	3	6.7366e-006	3.857e-006
4	2	1	2	3	4.3413e-006	7.0235e-006
5	2	2	3	1	5.3981e-006	9.5885e-006
6	2	3	1	2	9.4061e-006	1.594e-005
7	3	1	3	2	5.9087e-006	3.8218e-006
8	3	2	1	3	5.5662e-006	4.3919e-006
9	3	3	2	1	7.6662e-006	1.417e-005

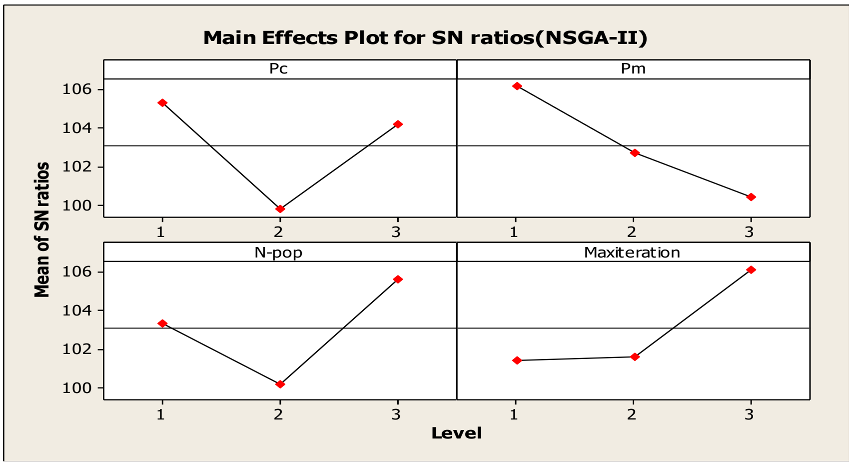

Fig. 8. Signal-noise diagram of the NSGA-II algorithm.

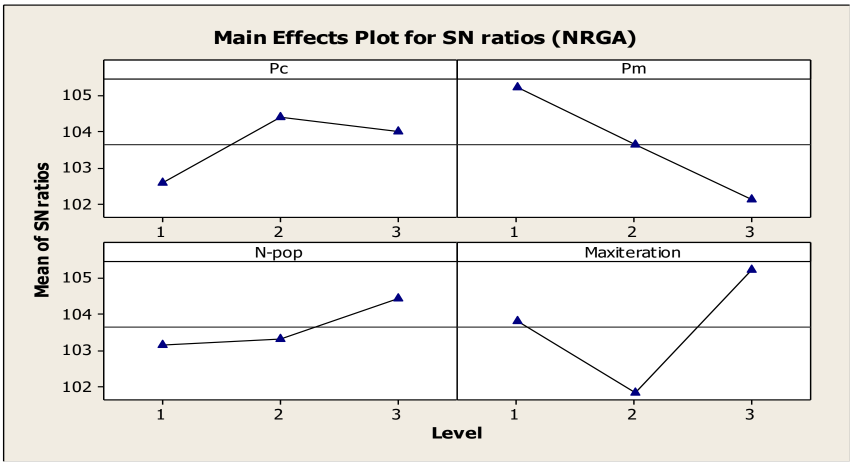

Fig. 9. Signal-noise diagram of the NRGA algorithm.

Following the design of the experiment and setting of the parameters, the appropriate parameters in each algorithm are now defined, and it is time to implement and compare the algorithms for the created problems.

Table 18. Computational results of the algorithms for 12 sub-problems.

Problem	NPS		CPU Time		MID	
	NSGA-II	NRGA	NSGA-II	NRGA	NSGA-II	NRGA
1	12	8	52.8815	58.4439	1.4909	2.3656
2	12	8	115.6659	129.4496	1.119	1.1781
3	15	14	199.8113	220.8354	2.1143	2.0267
4	8	12	302.7046	342.9649	3.6118	2.1146
5	11	7	746.2813	835.6521	3.6959	2.612
6	11	15	989.7469	1071.5596	3.1876	2.8049
7	6	7	1154.2441	1289.9778	5.0146	5.4399
8	10	15	1939.626	2179.4638	5.8759	5.6609
9	9	18	4644.4177	4215.4629	4.8438	4.797
10	17	19	5114.7147	4704.4016	3.9634	3.708
11	10	20	8779.6802	8592.3094	5.8276	4.0531
12	20	14	12039.6386	12803.8985	4.8701	6.3874

4.5 | Sensitivity Analysis

Fig. 10 to *Fig. 12* show the problem in various dimensions for a clearer understanding of certain Pareto charts.

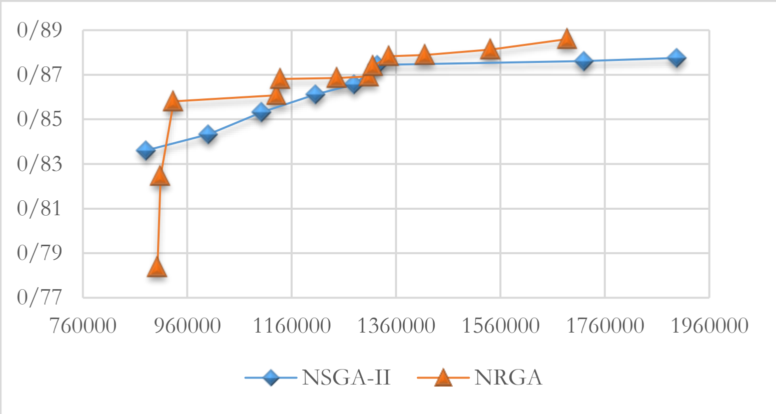

Fig. 10. Pareto chart for the low-dimensional (small-scale) problem.

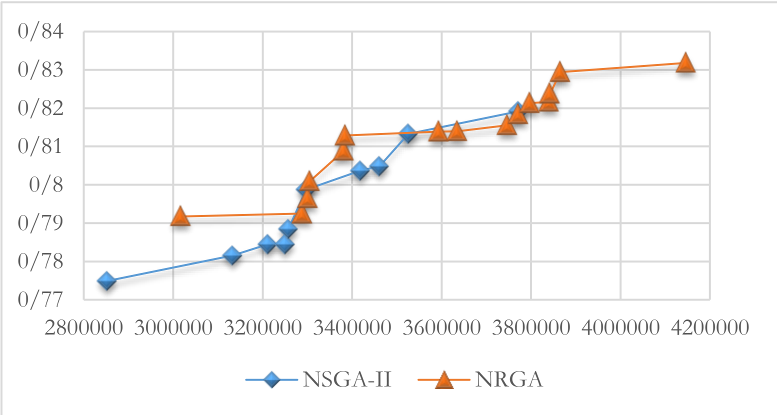

Fig. 11. Pareto chart for the mid-dimensional (medium-scale) problem.

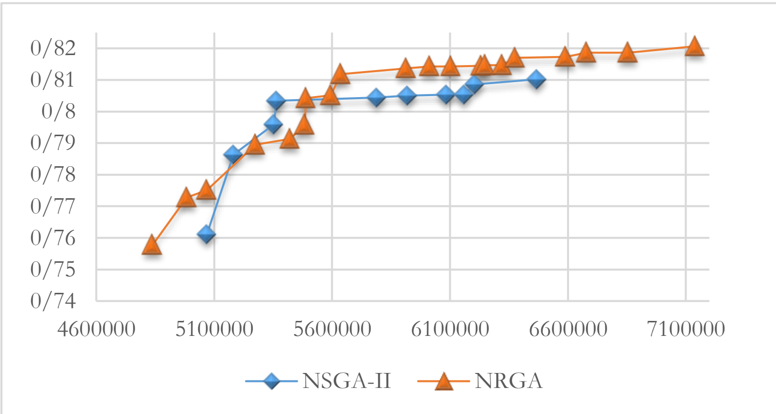

Fig. 12. Pareto chart for the high-dimensional (large-scale) problem.

According to the information provided:

NSGA-II was chosen as the best alternative on a small scale, followed by the NRGA algorithm. NRGA was chosen as the best alternative on a medium scale, followed by the NSGA-II algorithm. The NRGA algorithm was chosen as the best alternative on a large scale, followed by the NSGA-II algorithm.

5 | Managerial Insights

Natural disasters such as earthquakes, floods, hurricanes, and droughts strike various parts of the globe each year. Personal and financial losses are often associated with the occurrence of these natural disasters. Since the magnitude and scope of these disasters are often high, the demand for rescue operations is often uncertain. The number of relief centers available to meet the city's needs in normal conditions is often insufficient to meet demand at the appropriate time. Hence, the statistical fuzzy ideal programming model was presented in this study, which assesses the frequency of primary and secondary crises as well as accurately assesses and analyses the relief situation. Finally, using the epsilon-constraint method and metaheuristic algorithms NSGAII and NRGA, the proposed mathematical model was evaluated and analyzed to determine its validity. The findings showed that the above two algorithms are very effective in crisis management and significantly improve relief processes.

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A Method Based on the SCT to Solve the Inverse Problem for Heat Equation

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Abstract

In this article, a mathematical model of the inverse problem is considered. Based on this model a formulation of inverse problem for heat equation is proposed. Shifted Chebyshev Tau (SCT) method is suggested to solve the inverse problem. The aim of this determined effort is to identify unknown function and unknown control parameter of the mathematical model. In order to achieve highly accurate solution to this problem, the operational matrix of shifted Chebyshev polynomials is investigated in conjunction with tau scheme. To demonstrate the validity and applicability of the developed scheme, numerical example is presented.

Keywords: Inverse problem, Heat equation, Operational matrix, SCT method.

1 | Introduction

In this paper, a meticulous presentation is begun with an inverse problem arising in the heat equation, which aims to determine the function $u(x,t)$ and control parameter $P(t)$ in the following form [1]

$$u_t(x,t)=u_{xx}(x,t)+p(t)u(x,t)+q(x,t), \tag{1}$$

for $(x,t)\in(0,L)\times(0,\tau]$, where x and t represent space and time variables, respectively;

with initial condition

$$u(x,0)=f(x), \quad 0<x<L, \tag{2}$$

and boundary conditions

$$\alpha_1(t)u_x(0,t)+\beta_1(t)u(0,t)+\gamma_1(t)u(L,t)=g_1(t), \ 0<t\le\tau, \tag{3}$$

$$\alpha_2(t)u_x(L,t)+\beta_2(t)u(0,t)+\gamma_2(t)u(L,t)=g_2(t), \ 0<t\le\tau, \tag{4}$$

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and the energy condition

$$\int_0^{s(t)} u(x,t)dx = E(t), \quad 0 < t \le \tau, \ 0 < s(t) \le L, \tag{5}$$

where $q(x,t), f(x), E(t), \alpha_i(t), \beta_i(t), \gamma_i(t), g_i(t), i = 1,2$ are given functions.

Determination of an unknown control parameter is one of the hottest topics in inverse problems. There are many papers studying this type of equation [1]-[8]. Previously mentioned equation arise in many fields of science and engineering such as chemical diffusion, heat conduction processes, population dynamics, thermoelasticity, medical science, electrochemistry and control theory [9]-[15]. For example, microwave heat process used in various applications in industry, can be seen in ceramics and in food processing where the external energy is supplied to the target at a controlled level by the microwave-generating equipment. This can correspond to source term $p(t)u(x,t)$ in *Eq. (1)*, where $p(t)$ is proportional to power of external energy source and $u(x,t)$ is local conversion rate of microwave energy [12].

The energy *Condition (1)* is applied when the value of the control parameter $p(t)$ cannot be obtained using classical boundary conditions (which is divided into three phases: 1) neumann, 2) dirichlet, and 3) robin). Such type of condition can model various physical phenomena in context of heat transfer, life science and etc. [1], [2], [5], [6], [16], [17], [18]. The existence, uniqueness and continuous dependence of the solution upon the date for this problem are demonstrated in [1] under the following assumptions:

$$q \in C^{a,a/2}([0,L]\times[0,\tau]), \text{ for } 0 < a \le 1, \ f \in C^1[0,L], \ g_1, g_2 \in C[0,\tau],$$

$$E(0) = \int_0^{s(0)} f(x)dx > 0, \qquad E(t) > 0, s(t), E(t) \in C^1[0,\tau], \qquad \alpha_i, \beta_i, \gamma_i \in C[0,\tau],$$

$i = 1,2,$ and

$$\alpha_1(0)f_x(0) + \beta_1(0)f(0) + \gamma_1(0)f(L) = g_1(0),$$

$$\alpha_2(0)f_x(L) + \beta_2(0)f(0) + \gamma_2(0)f(L) = g_2(0).$$

Approximation and numerical solution of an inverse heat equation by control parameter are discussed in many papers, such as Boundary element method [8], finite volume element method [19], Generalized Fourier method [12], radial basis function collocation method [20], collocation method [21], Sinc-collocation method [17] and [6] third order compact Runge–Kutta method [8] and other mthods [3], [7], [21]-[27], must be used. This paper presents a simple and efficient algorithm for finding an approximate solution of *Eq. (1)* under the *Conditions (2)* to *(4)* and the energy *Condition (5)*. Instead, an algorithm which is called Shifted Chebyshev Tau (SCT), is proposed.

The main aim of this research is to use SCT method to solve an inverse heat *Eq. (5)*. Shifted Chebyshev polynomials of the first kind are put into practice to approximate the solution of the equation as a base of the tau method which is based on the shifted Chebyshev operational matrices of derivative and integration. The main advantage of this method is based upon reducing the PDE into a system of algebraic equation in the coefficient expansion of the solution. Numerical example, which confirm the accuracy of this method, is presented.

The presentation of this paper is as follows: a pair of transformations is brought to change the structure of the *Eqs. (1)* to *(5)*, then highlighting some necessary definitions and matrix formulation of Shifted Chebyshev polynomials, and construct its operational matrices of derivative and integral. In Section 3, the presented SCT method is used to find the approximate solution of the problem. As a result, a set of algebraic equations is formed and the solution of the considered problem is introduced. In Section 4, we discussed an error bound. Numerical results in Section 5 is given to show the efficiency of the proposed method. Finally, a brief conclusion is drawn in Section 6.

2 | Preparation and Foundation

At first, we that the pair of transformations constructed in follow:

$$r(t)=\exp\left(-\int_{o}^{t} p(s)ds\right), \tag{6}$$

$$v(x,t)=r(t)u(x,t). \tag{7}$$

The *Problems (1)* to *(5)* will become [5]:

$$v_t = v_{xx} + r(t)q(x,t), \qquad 0<x<L, 0<t\le\tau. \tag{8}$$

Subject to

$$v(x,0)=f(x), \qquad 0<x<L, \tag{9}$$

$$\alpha_1(t)v_x(0,t)+\beta_1(t)v(0,t)+\gamma_1(t)v(L,t)=r(t)g_1(t), \qquad 0<t\le\tau, \tag{10}$$

$$\alpha_2(t)v_x(L,t)+\beta_2(t)v(0,t)+\gamma_2(t)v(L,t)=r(t)g_2(t), \qquad 0<t\le\tau, \tag{11}$$

and

$$\int_0^{s(t)} v(x,t)dx = r(t)E(t), \qquad 0<t\le\tau,\ 0<s(t)<L. \tag{12}$$

Obviously, if we have $v(x,t)$ and $r(t)$ then by using *Eqs. (8)* to *(12)*, $u(x,t)$ and $p(t)$ can be found as:

$$u(x,t)=\frac{v(x,t)}{r(t)}, \qquad 0<x<L, 0<t\le\tau, \tag{13}$$

$$p(t)=-\frac{r'(t)}{r(t)}, \qquad 0<t\le\tau. \tag{14}$$

In this transformation, the source parameter disappeared, so we can solve the *Eqs. (8)* to *(12)*.

2.1 | Basic Definitions and Matrix Formulation

In this section, some fundamental definitions are given and to introduce the necessary notation, also matrix formulation of Shifted Chebyshev polynomial of the first kind which will be used throughout the paper.

The shifted Chebyshev polynomials are generated from the following three-term recurrence relation:

$$\begin{aligned} &T_{L,0}(x)=1, T_{L,1}(x)=\frac{2x}{L}-1, \\ &T_{L,j}(x)=2\left(\frac{2x}{L}-1\right)T_{L,j-1}(x)-T_{L,j-2}(x), \quad j=2,3,\dots,n. \end{aligned} \tag{15}$$

Definition 1. Let $T_{L,j}(x)$ imply the shifted Chebyshev polynomial of the order *j* then $T_{L,j}(x)$ can be formulated as [22] and [25].

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$$T_{L,j}(x)=j\sum_{k=0}^{j}(-1)^{j-k}\frac{(j+k-1)!2^{2k}}{(j-k)!(2k)!L^{k}}x^{k},\quad j=1,2,3,\ldots,n, \tag{16}$$

where $T_{L,j}(0)=(-1)^{j}$ and $T_{L,j}(L)=1$. The orthogonality condition is:

$$\int_{0}^{L}T_{L,j}(x)T_{L,k}(x)w_{L}(x)dx=h_{j}, \tag{17}$$

where the weight function

$$w_{L}(x)=\frac{1}{\sqrt{Lx-x^{2}}}, \tag{18}$$

and

$$h_{j}=\begin{cases}\frac{\varepsilon_{j}}{2}\pi & ,\ k=j,\\ 0 & ,\ k\neq j,\end{cases}\quad \varepsilon_{0}=2,\varepsilon_{j}=1;j\geq 1. \tag{19}$$

Definition 2. Let $v(x,t)$ be function defined for $0<x<L,0<t\leq\tau$ and then expanded in the terms of the shifted Chebyshev polynomial as [22] and [24]:

$$v(x,t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}a_{ij}T_{\tau,i}(t)T_{L,j}(x). \tag{20}$$

If the infinite series in *Eq. (20)* is truncated, then the function $v(x,t)$ can be approximated as:

$$v_{m,n}(x,t)\simeq\sum_{i=0}^{m}\sum_{j=0}^{n}a_{ij}T_{\tau,i}(t)T_{L,j}(x)=\psi^{T}(t)Af(x), \tag{21}$$

where the shifted Chebyshev vectors $\psi(t)$ and $f(x)$ and the matrix A are given as:

$$\psi(t)=\left[T_{\tau,0}(t),T_{\tau,1}(t),\ldots,T_{\tau,m}(t)\right]^{T},$$
$$\boldsymbol{f}(x)=\left[T_{L,0}(x),T_{L,1}(x),\ldots,T_{L,n}(x)\right]^{T},$$

$$A=\begin{bmatrix}a_{00} & a_{01} & \cdots & a_{0n}\\ a_{10} & a_{11} & \cdots & a_{1n}\\ \vdots & \vdots & \cdots & \vdots\\ a_{m0} & a_{m1} & \cdots & a_{mn}\end{bmatrix}. \tag{22}$$

Here, the shifted Chebyshev coefficient matrix $A=(a_{ij})$ is given by

$$a_{ij}=\frac{1}{h_{i}h_{j}}\int_{0}^{\tau}\int_{0}^{L}u(x,t)T_{\tau,i}(t)T_{L,j}(x)w_{\tau}(t)w_{L}(x)dxdt,\quad i=0,1,\ldots,m,\ j=0,1,\ldots,n. \tag{23}$$

We approximate functions $v(x,t)$, $q(x,t)$ and $f(x)$ by using the shifted Chebyshev operational matrix follow as:

$$
\begin{cases}
v_{m,n}(x,t)=\psi^{T}(t)A\mathfrak{f}(x), \\
q_{m,n}(x,t)\simeq\sum_{i=0}^{m}\sum_{j=0}^{n}q_{ij}T_{\tau,i}(t)T_{L,j}(x)=\psi^{T}(t)Q\mathfrak{f}(x), \\
f(x)\simeq\sum_{j=0}^{n}f_{j}T_{L,j}(x)=\psi^{T}(t)F\mathfrak{f}(x),
\end{cases}
\tag{24}
$$

where A is an unknown $(m+1)\times(n+1)$ matrix, Q and F are known $(m+1)\times(n+1)$ matrices as:

$$
Q=\begin{bmatrix} q_{00} & q_{01} & \cdots & q_{0n} \\ q_{01} & q_{11} & \cdots & q_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ q_{m0} & q_{m1} & \cdots & q_{mn} \end{bmatrix},\quad F=\begin{bmatrix} f_0 & f_1 & \cdots & f_{n-1} & f_n \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix},
\tag{25}
$$

where

$$
q_{ij}=\frac{1}{h_i h_j}\int_0^{\tau}\int_0^{L}q(x,t)T_{\tau,i}(t)T_{L,j}(x)w_{\tau}(t)w_{L}(x)dxdt,\quad i=0,1,\ldots,m,\ j=0,1,\ldots,n,
\tag{26}
$$

and

$$
f_j=\frac{1}{h_j}\int_0^{L}f(x)T_{L,j}(x)w_{L}(x)dx,\quad j=0,1,\ldots,n.
\tag{27}
$$

2.2 | Operational Matrices of Derivative and Integral

In this section, Shifted Chebyshev Vectors are used and so as its operational matrices of derivative and integral to solve inverse heat problem of the form *Eqs. (8)* to *(12)*.

Theorem 1. The derivative of the shifted Chebyshev vector $\mathfrak{f}(x)$ may be expressed by [1], [2], [4], [24].

$$
\frac{d\mathfrak{f}(x)}{dx}=D^{(1)}\mathfrak{f}(x),
\tag{28}
$$

where $D^{(1)}=d_{ij}$ is the $(n+1)\times(n+1)$ operational matrix of derivative and given by

$$
D^{(1)}=d_{ij}=\begin{cases} \dfrac{4i}{\varepsilon_j L}, & j=i-k, \begin{cases} k=1,3,\ldots,n, & \text{if}(n)\text{isodd}, \\ k=1,3,\ldots,n-1, & \text{if}(n)\text{iseven}, \end{cases} \\ 0, & \text{otherwise}. \end{cases}
\tag{29}
$$

where $\varepsilon_0=2,\varepsilon_j=1,j\geq 1$, see [4] and [24].

Corollary 1. Using *Eq. (28)*, the operational matrix for the nth derivative can be stated as [2], [3], [25].

$$
\frac{d^n\phi(x)}{dx^n}=\left(D^{(1)}\right)^n\phi(x),
\tag{30}
$$

where $n\in N$ is the nth power of matrix $D^{(1)}$. So we have

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$$D^{n}=\left(D^{(1)}\right)^{n}, n=1,2,.... \tag{31}$$

Theorem 2. The integration of $\psi_{\tau,m}(t)$ may be written as [2], [3], [25].

$$\int_{0}^{t}\psi(t')dt'\simeq P\psi(t), \tag{32}$$

where P is the $(m+1)\times(m+1)$ shifted Chebyshev operational matrix of integration and is given by

$$p=\begin{bmatrix} w_0 & \delta_0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ w_1 & 0 & \lambda_1 & 0 & 0 & \cdots & 0 & 0 \\ w_2 & \delta_2 & 0 & \lambda_2 & 0 & \cdots & 0 & 0 \\ w_3 & 0 & \delta_3 & 0 & \lambda_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ w_{m-2} & 0 & 0 & 0 & \ddots & \ddots & \lambda_{m-2} & 0 \\ w_{m-1} & 0 & 0 & 0 & 0 & \ddots & 0 & \lambda_{m-1} \\ w_m & 0 & 0 & 0 & 0 & \cdots & \delta_m & 0 \end{bmatrix}, \tag{33}$$

where w_k, δ_k and λ_k obtained using the following formula:

$$w_k=\begin{cases} \frac{\tau}{2}, & k=0, \\ \frac{-\tau}{8}, & k=1, \\ \frac{(-1)^{k+1}\tau}{2(k-1)(k+1)}, & k=2,3,... \end{cases}, \quad \delta_k=\begin{cases} \frac{\tau}{2}, & k=0, \\ 0, & k=1, \\ \frac{-\tau}{4(k-1)}, & k=2,3,... \end{cases}, \tag{34}$$

$$\lambda_k=\begin{cases} 0, & k=0, \\ \frac{\tau}{8}, & k=1, \\ \frac{\tau}{4(k+1)}, & k=2,3,... \end{cases}$$

Obviously similar to *Eq. (32)* we have

$$\int_{0}^{x}f(x')dx'\simeq Gf(x), \tag{35}$$

where G is the $(n+1)\times(n+1)$ shifted Chebyshev operational matrix of integration and is defined similar to *Eq. (33)*.

3 | Shifted Chebyshev Tau Method

In this part, SCT method is applicable to solve the inverse problem for heat *Eqs. (8)* to *(12)*.

Integrating *Eq. (8)* from 0 to t and using *Eq. (9)*, we have

$$v(x,t)-f(x)=\int_{o}^{t}v_{xx}(x,t')dt'+\int_{o}^{t}r(t')q(x,t')dt'. \tag{36}$$

Using *Eq. (24)*, *Corollary 1* and *Theorem 2* we obtain

$$\int_0^t v_{xx}(x,t')dt' = \left(\int_0^t \psi^T(t')dt'\right)A\left(\frac{d^2\mathfrak{f}(x)}{dx^2}\right) = \psi^T(t)P^T AD^2\mathfrak{f}(x). \tag{37}$$

The function $r(t)$ may be expanded in terms of $m+1$ shifted Chebyshev series as (see [3] and [24]).

$$r(t) \simeq r_m(t) = \sum_{k=0}^{m} b_k T_{\tau,k}(t) = B^T\psi(t), \tag{38}$$

where $B = [b_0, b_1, ..., b_m]^T$ is an unknown vector.

Now, using *Eqs. (19)*, *(30)* and *(38)* we have

$$\int_0^t r(t')q(x,t')dt' = \left(\int_0^t B^T\psi(t')\psi^T(t')dt'\right)Q\mathfrak{f}(x). \tag{39}$$

Let us set

$$B^T\psi(t)\psi^T(t) = \psi^T(t)H, \tag{40}$$

where H is an $(m+1)\times(m+1)$ matrix. To find H, we rewrite *Eq. (40)* (see [2] and [3]) in the form

$$\sum_{k=0}^{m} b_k T_{\tau,k}(t)T_{\tau,j}(t) = \sum_{k=0}^{m} H_{kj}T_{\tau,k}(t), \qquad j = 0,1,...,m. \tag{41}$$

Multiplying both sides of *Eq. (41)* by $T_{\tau,i}(t)w_\tau(t)$, $i = 0,1,...,m$ and integrating from 0 to τ yields

$$\sum_{k=0}^{m} b_k \int_0^\tau T_{\tau,i}(t)T_{\tau,k}(t)T_{\tau,j}(t)w_\tau(t)dt = \sum_{k=0}^{m} H_{kj}\int_0^\tau T_{\tau,k}(t)T_{\tau,i}(t)w_\tau(t)dt. \tag{42}$$
$$i,j = 0,1,...,m.$$

Byusing *Eq. (42)* and employing the orthogonality relation *Eq. (8)* gives:

$$\sum_{k=0}^{m} b_k \int_0^\tau T_{\tau,i}(t)T_{\tau,k}(t)T_{\tau,j}(t)w_\tau(t)dt = H_{ij}h_i,$$

or equivalently

$$H_{ij} = \frac{1}{h_i}\sum_{k=0}^{m} b_k \int_0^\tau T_{\tau,i}(t)T_{\tau,k}(t)T_{\tau,j}(t)w_\tau(t)dt, \qquad i,j = 0,1,...,m. \tag{43}$$

Employing *Eqs. (32), (39)* and *(40)* can be written as:

$$\int_0^t r(t')q(x,t')dt' = \psi^T(t)P^T HQ\mathfrak{f}(x). \tag{44}$$

Applying *Eqs. (21), (24), (37)* and *(44)* the residual $Res_{m,n}(x,t)$ for *Eq. (36)* can be written as:

$$Res_{m,n}(x,t) = \psi^T(t)\left[A - F - P^T HQ - P^T AD^2\right]\mathfrak{f}(x).$$

Employing standard tau method, generate $(m+1)\times(n-1)$ linear algebraic equations using the following algebraic equations:

$$\int_0^\tau\int_0^L Res_{m,n}(x,t)T_{\tau,i}(t)T_{L,j}(x)dxdt = 0, \qquad i = 0,1,...,m, \; j = 0,1,...,n-2. \tag{45}$$

Also, by substituting *Eqs. (24)* and *(38)* in *Eqs. (10)* and *(11)* we get

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$$\begin{aligned}&\alpha_1(t)\psi^T(t)ADf(0)+\beta_1(t)\psi^T(t)Af(0)+\gamma_1(t)\psi^T(t)Af(L)=g_1(t)\psi^T(t)B,\\&\alpha_2(t)\psi^T(t)ADf(L)+\beta_2(t)\psi^T(t)Af(0)+\gamma_2(t)\psi^T(t)Af(L)=g_2(t)\psi^T(t)B.\end{aligned}\tag{46}$$

And applying *Eqs. (24)* and *(35)* in *Eq. (12)* we have

$$\psi^T(t)AGf(s(t))=E(t)\psi^T(t)B.\tag{47}$$

Eqs. (46) to *(47)* are collocated at $m+1$ points. Here, the roots of $T_{\tau,m+1}(t)$ are used as a collocation points. *Eqs. (45)* to *(47)* yields a set of $(m+1)(n+1)+(m+1)$ algebraic equations which can be solved for $a_{ij}, i=0,1,\dots,m, j=0,1,\dots,n$ and $b_k, k=0,1,\dots,m$. Consequently $v(x,t)$ given in *Eq. (21)* and $r(t)$ given in *Eq. (38)* can be calculated. Finally using *Eqs. (13)* and *(14)*, $u(x,t)$ and $p(t)$ can be found.

4 | Error Bound

In this section, an upper bound of the absolute errors will be given by using Lagrange interpolation polynomials. Our aim is to obtain an analytic expression for the error of the best approximation of a smooth function $v(x,t)$ and source function $r(t)$ by them expansion in terms of shifted Chebyshev polynomials.

Theorem 3. If $v(x,t)\in\Omega\equiv[0,L]\times[0,\tau]$ is a sufficiently smooth function and $v_{m,n}(x,t)$ is the interpolating polynomial for $v(x,t)$ at points (x_j,t_i) where x_j , $0\le j\le n$ are the roots of the $T_{L,j}(x)$ in $[0,L]$ and t_i, $0\le i\le m$ are the roots of the $T_{\tau,i}(t)$ in $[0,\tau]$, then the error bound is presented as follows:

$$\left|v(x,t)-v_{m,n}(x,t)\right|\le K_1\frac{\left(\frac{L}{2}\right)^{n+1}}{(n+1)!2^n}+K_2\frac{\left(\frac{\tau}{2}\right)^{m+1}}{(m+1)!2^m}+K_3\frac{\left(\frac{L}{2}\right)^{n+1}\left(\frac{\tau}{2}\right)^{m+1}}{(n+1)!(m+1)!2^{m+n}}.$$

Proof: Let us define the error function $v(x,t)-v_{m,n}(x,t)$, then by similar procedures as in [26], we have

$$\begin{aligned}v(x,t)-v_{m,n}(x,t)=&\frac{\partial^{n+1}v(\xi,t)}{\partial x^{n+1}(n+1)!}\prod_{j=0}^{n}(x-x_j)+\frac{\partial^{m+1}v(x,\eta)}{\partial t^{m+1}(m+1)!}\prod_{i=0}^{m}(t-t_i)-\\&\frac{\partial^{m+n+2}v(\xi',\eta')}{\partial x^{n+1}\partial t^{m+1}(m+1)!(n+1)!}\prod_{j=0}^{n}(x-x_j)\prod_{i=0}^{m}(t-t_i),\end{aligned}\tag{48}$$

where $\xi,\xi'\in[0,L]$ and $\eta,\eta'\in[0,\tau]$. Therefore

$$\begin{aligned}\left|v(x,t)-v_{m,n}(x,t)\right|=&\max_{(x,t)\in\Omega}\left|\frac{\partial^{n+1}v(x,t)}{\partial x^{n+1}}\right|\frac{\prod_{j=0}^{n}\left|x-x_j\right|}{(n+1)!}+\max_{(x,t)\in\Omega}\left|\frac{\partial^{m+1}v(x,t)}{\partial t^{m+1}}\right|\frac{\prod_{i=0}^{m}\left|t-t_i\right|}{(m+1)!}-\\&\max_{(x,t)\in\Omega}\left|\frac{\partial^{m+n+2}v(x,t)}{\partial x^{m+1}\partial t^{n+1}}\right|\frac{\prod_{j=0}^{n}\left|x-x_j\right|\prod_{i=0}^{m}\left|t-t_i\right|}{(m+1)!(n+1)!}.\end{aligned}\tag{49}$$

Assume that there are constants K_1 , K_2 and K_3 , such that

$$\max_{(x,t)\in\Omega}\left|\frac{\partial^{n+1}v(x,t)}{\partial x^{n+1}}\right|\le K_1,$$
$$\max_{(x,t)\in\Omega}\left|\frac{\partial^{m+1}v(x,t)}{\partial t^{m+1}}\right|\le K_2, \tag{50}$$
$$\max_{(x,t)\in\Omega}\left|\frac{\partial^{n+m+2}v(x,t)}{\partial x^{n+1}\partial t^{m+1}}\right|\le K_3.$$

Let us use the one-to-one mapping $x = 2L(z+1)$ between the intervals $[-1,1]$ and $[0,L]$ to deduce and also taking into account the estimates for Chebyshev interpolation nodes, then we obtain:

$$\begin{aligned}\min_{x_j\in[0,L]}\max_{0\le x\le L}\left|\prod_{j=0}^{n}(x-x_j)\right| &= \min_{z_j\in[-1,1]}\max_{-1\le z\le 1}\left|\prod_{j=0}^{n}\frac{L}{2}(z-z_j)\right| \\ &=\left(\frac{L}{2}\right)^{n+1}\min_{z_j\in[-1,1]}\max_{-1\le z\le 1}\left|\prod_{j=0}^{n}(z-z_j)\right| = \left(\frac{L}{2}\right)^{n+1}\frac{1}{2^n}.\end{aligned} \tag{51}$$

Now, by replacing *Eqs. (50)* and *(51)* in *Eq. (49)*, yields the following desired result:

$$\max_{(x,t)\in\Omega}\left|\frac{\partial^{n+1}v(x,t)}{\partial x^{n+1}}\right|\le K_1,$$
$$\max_{(x,t)\in\Omega}\left|\frac{\partial^{n+m+2}v(x,t)}{\partial x^{n+1}\partial t^{m+1}}\right|\le K_3, \tag{52}$$

or equivalently

$$\left|v(x,t)-v_{m,n}(x,t)\right|\le K_1\frac{L^{n+1}}{(n+1)!2^{2n+1}}+K_2\frac{\tau^{m+1}}{(m+1)!2^{2m+1}}+K_3\frac{L^{n+1}\tau^{m+1}}{(n+1)!(m+1)!2^{2m+2n+1}}. \tag{53}$$

Therefore, an upper bound of the absolute errors is obtained for the approximate and exact solutions.

Remark 1. In the special case if $m = n$ and $L = \tau = 1$ we have

$$\left|v(x,t)-v_{n,n}(x,t)\right|\le\left(K_1+K_2+\frac{K_3}{(n+1)!2^{2n+1}}\right)\frac{1}{(n+1)!2^{2n+1}}. \tag{54}$$

Let

$$K_1+K_2+\frac{K_3}{(n+1)!2^{2n+1}}=\alpha. \tag{55}$$

So that we may write

$$\left|v(x,t)-v_{n,n}(x,t)\right|\le\frac{\alpha}{(n+1)!2^{2n+1}}, \tag{56}$$

Hence, show that

$$\left|v(x,t)-v_{n,n}(x,t)\right|=O\left(\frac{1}{(n+1)!2^{2n+1}}\right). \tag{57}$$

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Theorem 4. Let $r_m(t)=\sum_{k=0}^{m} b_k T_{\tau,k}(t)$, be the shifted chebyshev functions expansion of the real sufficiently smooth function $r(t)$ and there is a real number K such that

$$\left\| r(t)-r_m(t) \right\|_2 \le K\frac{\tau^{m+1}}{(m+1)!2^{2m+1}}. \tag{58}$$

Moreover, if $\bar{r}_m(t)=\sum_{k=0}^{m} \bar{b}_k T_{\tau,k}(t)$ be an approximation for the shifted chebyshev functions $r_m(t)$, then there are μ_τ and λ_τ such that

$$\left\| r(t)-\bar{r}_m(t) \right\|_2 \le \mu_\tau\left(\frac{K\tau^{m+1}}{2^{2m+1}(m+1)!}\right)+\lambda_\tau\left\| B-\bar{B} \right\|_2. \tag{59}$$

Proof: To prove *Eq. (59)*, we write

$$r(t)-\bar{r}_m(t)=r(t)-r_m(t)+r_m(t)-\bar{r}_m(t), \tag{60}$$

Satisfies the triangle inequality

$$\left\| r(t)-\bar{r}_m(t) \right\|_2 \le \left\| r(t)-r_m(t) \right\|_2+\left\| r_m(t)-\bar{r}_m(t) \right\|_2, \tag{61}$$

The right-hand inequality in *Eq. (61)* write as follow:

$$\begin{aligned} \left\| r(t)-r_m(t) \right\|_2 &=\left(\int_0^\tau w_\tau(t)\left|r(t)-r_m(t)\right|^2 dt\right)^{\frac{1}{2}} \le \left(\int_0^\tau w_\tau(t)\left(\frac{K\tau^{m+1}}{2^{2m+1}(m+1)!}\right)^2 dt\right)^{\frac{1}{2}} \\ &=\left(\frac{K\tau^{m+1}}{2^{2m+1}(m+1)!}\right)\left(\int_0^\tau w_\tau(t)dt\right)^{\frac{1}{2}}=\mu_\tau\left(\frac{K\tau^{m+1}}{2^{2m+1}(m+1)!}\right), \end{aligned} \tag{62}$$

where

$$\mu_\tau=\frac{1}{4}\sqrt{t-t^2}(2t-1)-\sin^{-1}\left(\sqrt{1-t}\right).$$

Moreover

$$\begin{aligned} \left\| r_m(t)-\bar{r}_m(t) \right\|_2 &=\left(\int_0^\tau w_\tau(t)\left[\sum_{k=0}^{m}(b_k-\bar{b}_k)T_{\tau,k}(t)\right]^2 dt\right)^{\frac{1}{2}} \\ &\le\left(\int_0^\tau w_\tau(t)\left[\sum_{k=0}^{m}\left|c_i-\bar{c}_i\right|^2\right]\left[\sum_{k=0}^{m}\left|T_{\tau,k}(t)\right|^2\right]dt\right)^{\frac{1}{2}} \\ &=\left(\sum_{k=0}^{m}\left|b_k-\bar{b}_k\right|^2\right)^{\frac{1}{2}}\left(\int_0^\tau w_\tau(t)\left[\sum_{k=0}^{m}\left|T_{\tau,k}(t)\right|^2\right]dt\right)^{\frac{1}{2}}=\left\|B-\bar{B}\right\|_2\left(\sum_{k=0}^{m}h_{\tau,k}\right)^{\frac{1}{2}}=\lambda_\tau\left\|B-\bar{B}\right\|_2, \end{aligned} \tag{63}$$

where

$$\lambda_\tau=\left(\sum_{k=0}^{m}h_{\tau,k}\right)^{\frac{1}{2}}.$$

By summing relation *Eqs. (62)* and *(63)* upper-bound in *Theorem 4* the following relation is created:

$$\left\| r(t)-\bar{r}_m(t) \right\|_2 \le \mu_\tau\left(\frac{K\tau^{m+1}}{2^{2m+1}(m+1)!}\right)+\lambda_\tau\left\| B-\bar{B} \right\|_2.$$

This completes the proof.

5 | Numerical Results

In this section, numerical experiment is chosen to illustrate the efficiency and performance of the SCT method for solving *Eq. (1)* with *Conditions (1)* to *(4)* and energy *Condition (5)*. In this case, the exact solution of the problem is known. The accuracy of our approach is estimated by the following error functions:

$$e_u = \left|u_{m,n}(x,t) - u(x,t)\right|, \quad e_p = \left|p_m(t) - p(t)\right|.$$

Example 1. *Considers (1)* to *(5)* with the given data:

$$\tau = 0.5, L = 1,$$
$$q(x,t) = (1-t^3)\sin x - x^2(t-1)^2\exp(t^2) - 2\exp(t^2) - t^2(\pi\cos x + t^3 + t - 3),$$
$$f(x) = x^2 + \pi\cos x,$$
$$g_1(t) = \pi + t^3,$$
$$g_2(t) = t\sin 1 + \exp(t^2) + \pi\cos 1 + t^3,$$
$$\alpha_1 = 0, \alpha_2 = 0,$$
$$\beta_1(t) = 1, \beta_2(t) = 0,$$
$$\gamma_1(t) = 0, \gamma_2(t) = 1,$$
$$E(t) = (\pi\sin t - t\cos t)\cos(\sin t) + (t\sin t + \pi\cos t)\sin(\sin t) +$$
$$\frac{1}{3}\exp(t^2)(\sin^3 t + t^3) + (t^2\sin t + t\sin^2 t)\exp(t^2) + (1 + t^3 + t^2\sin t)t,$$
$$s(t) = t + \sin t.$$

For which the exact solution is [27]

$$u(x,t) = t\sin x + x^2\exp(t^2) + \pi\cos x + t^3,$$

and

$$p(t) = 1 + t^2.$$

In *Table 1*, we display error function $\left|u_{m,n}(x,t) - u(x,t)\right|$, using the proposed method at t = 0.25 with m = n = 4, 6, 8. Also, the results obtained for $\left|p_m(t) - p(t)\right|$ are listed in *Table 2*. In *Fig. 1*, the space-time graph of exact solution $u(x,t)$ and time graph of $p(t)$ are plotted. In *Fig. 2* and *Fig. 3* graph of the absolute error $u(x,0.25)$ for x=0.1 and x=0.9 with various value of m=n and $\left|p_m(t) - p(t)\right|$ for t=0.1 and t=0.5 with various value of m are shown respectively. *Fig. 4* and *Fig. 5* Graph of the absolute error for $u(x,0.25)$ with m=n=4, 6, 8 and $\left|p_m(t) - p(t)\right|$ for m=n=4, 6, 8 obtained.

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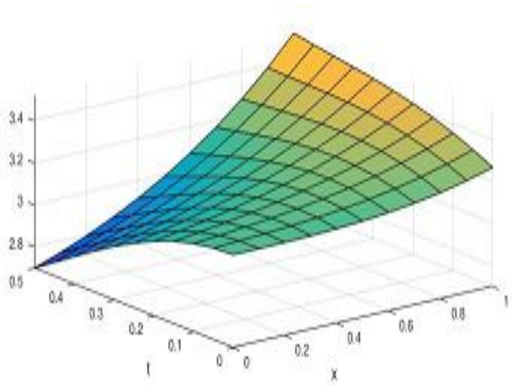
Table 1. Error function $\left|u_{m,n}(x,t)-u(x,t)\right|$ with t = 0.25.

x	Error $m=n=4$	$m=n=6$	$m=n=8$
0.1	7.33×10^{-5}	1.88×10^{-6}	2.25×10^{-8}
0.2	4.85×10^{-5}	7.52×10^{-9}	2.50×10^{-10}
0.3	1.15×10^{-5}	1.44×10^{-7}	5.64×10^{-10}
0.4	6.99×10^{-5}	1.56×10^{-6}	1.87×10^{-8}
0.5	5.74×10^{-4}	3.95×10^{-5}	6.68×10^{-6}
0.6	2.31×10^{-4}	4.01×10^{-6}	3.54×10^{-8}
0.7	1.89×10^{-4}	1.99×10^{-6}	2.15×10^{-8}
0.8	5.49×10^{-5}	1.78×10^{-7}	5.47×10^{-9}

Table 2. Error function $\left|p_m(t)-p(t)\right|$.

t	Error $m=n=4$	$m=n=6$	$m=n=8$
0.05	2.29×10^{-3}	5.76×10^{-5}	6.99×10^{-7}
0.1	2.60×10^{-4}	4.33×10^{-6}	1.34×10^{-8}
0.15	3.21×10^{-3}	4.81×10^{-5}	2.57×10^{-7}
0.2	8.10×10^{-4}	1.81×10^{-5}	2.07×10^{-7}
0.25	3.37×10^{-3}	2.12×10^{-4}	3.92×10^{-6}
0.3	1.09×10^{-3}	1.99×10^{-6}	1.67×10^{-8}
0.35	8.69×10^{-3}	9.08×10^{-5}	9.77×10^{-7}
0.4	3.04×10^{-3}	9.05×10^{-6}	3.93×10^{-7}
0.45	4.08×10^{-3}	9.14×10^{-5}	3.01×10^{-5}
0.5	4.12×10^{-3}	9.84×10^{-5}	3.76×10^{-5}



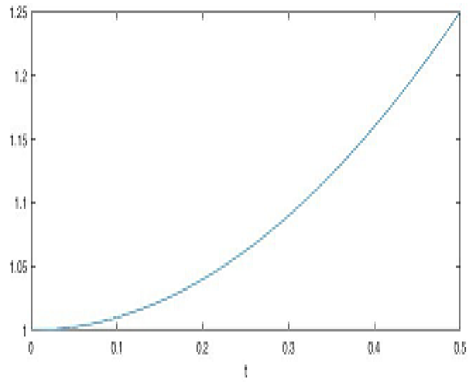


Fig. 1. The space-time graph of exact solution u(x,t) (left) and time graph of $p(t)$ (right).

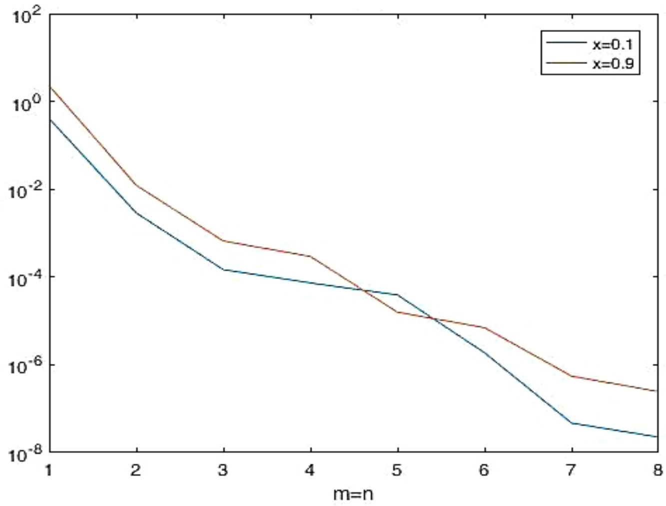


Fig. 2. Graph of the absolute error u(x, 0.025) for x=0.1 and x=0.9 with various value of m=n.

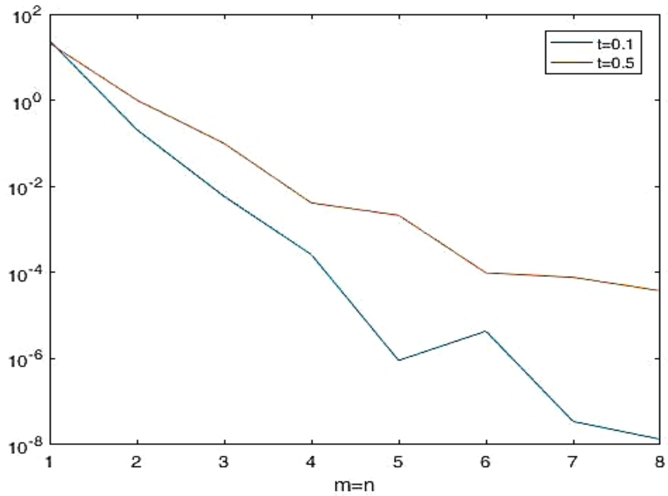


Fig. 3. Plot of $|p_m(t) - p(t)|$ for t=0.1 and t=0.5 with various value of m.

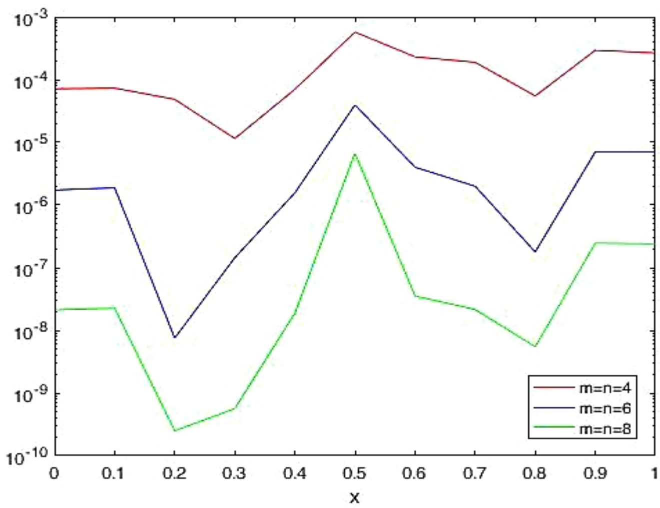


Fig. 4. Graph of the absolute error for u(x, 0.25) with m=n=4, 6, 8.

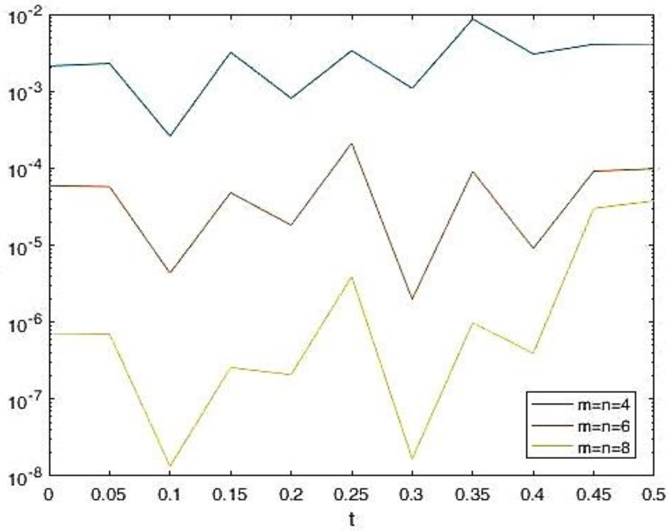


Fig. 5. Plot of $|p_m(t) - p(t)|$ with m=4, 6, 8.

Example 2. We consider the second inverse *Problems (1)* to *(5)* with;

$$\begin{aligned}
&\tau = 1, L = 1\\
&q(x,t) = 0,\\
&f(x) = 1 + \cos x,\\
&g_1(t) = t^2\left(1 + e^{-t}\sin 1\right)\exp\left(t^2 - \sin t\right),\\
&g_2(t) = \left(t\left(1 + e^{-t}\cos 1\right) - e^{-t}\sin 1\right)\exp\left(t^2 - \sin t\right),\\
&\alpha_1 = 1, \alpha_2 = 1,\\
&\beta_1(t) = t^2, \beta_2(t) = 0,\\
&\gamma_1(t) = 0, \gamma_2(t) = 1,\\
&E(t) = \left(1 + e^{-t}\sin 1\right)\exp\left(t^2 - \sin t\right),\\
&s(t) = 1.
\end{aligned}$$

For which the exact solution is [17]:

$$u(x,t) = \left(1 + e^{-t}\cos x\right)\exp\left(t^2 - \sin t\right),$$

and

$$p(t) = 2t - \cos t.$$

In *Table 3*, we display error function $|u_{m,n}(x,t) - u(x,t)|$, using the proposed method at $t = 0.5$ with $m = n = 4, 6, 8$. Also, the results obtained for $|p_m(t) - p(t)|$ are listed in *Table 4*. In *Fig. 6*, the space-time graph of exact solution $u(x,t)$ and time graph of $p(t)$ are plotted. In *Fig. 7* and *Fig. 8* Graph of the absolute error $u(x,0.5)$ for x=0.1 and x=0.9 with various value of m=n and $|p_m(t) - p(t)|$ for t=0.1 and t=1 with various value of m are shown respectively. *Fig. 9* and *Fig. 10* graph of the absolute error for $u(x,0.5)$ with m=n=4, 6, 8 and $|p_m(t) - p(t)|$ for m=n=4, 6, 8 obtained.

Table 3. Error function $|u_{m,n}(x,t) - u(x,t)|$ with $t = 0.5$.

	Error		
x	$m=n=4$	$m=n=6$	$m=n=8$
0.1	1.49×10^{-4}	3.82×10^{-6}	4.58×10^{-8}
0.2	9.83×10^{-5}	1.53×10^{-8}	5.1×10^{-10}
0.3	7.38×10^{-5}	9.23×10^{-7}	3.69×10^{-9}
0.4	1.07×10^{-4}	2.38×10^{-6}	2.86×10^{-8}
0.5	2.22×10^{-4}	1.53×10^{-5}	2.59×10^{-6}
0.6	2.17×10^{-5}	3.77×10^{-7}	3.32×10^{-9}
0.7	-8.76×10^{-5}	-9.21×10^{-7}	-9.95×10^{-9}
0.8	-8.44×10^{-6}	-2.74×10^{-8}	-8.41×10^{-10}
0.9	-9.25×10^{-5}	-2.22×10^{-6}	-7.77×10^{-8}

Table 4. Error function $|p_m(t) - p(t)|$.

	Error		
t	$m=n=4$	$m=n=6$	$m=n=8$
0.1	7.48×10^{-4}	1.17×10^{-5}	2.24×10^{-7}
0.2	3.80×10^{-4}	7.47×10^{-8}	2.49×10^{-9}
0.3	3.73×10^{-4}	4.48×10^{-6}	1.79×10^{-8}
0.4	3.39×10^{-4}	4.20×10^{-5}	1.44×10^{-7}
0.5	1.48×10^{-3}	7.43×10^{-5}	1.26×10^{-8}
0.6	3.04×10^{-4}	5.80×10^{-6}	1.19×10^{-8}
0.7	4.27×10^{-4}	3.49×10^{-6}	4.48×10^{-8}
0.8	4.12×10^{-5}	1.34×10^{-7}	4.11×10^{-9}
0.9	4.14×10^{-4}	1.49×10^{-5}	5.81×10^{-7}
1	4.84×10^{-4}	1.38×10^{-5}	4.53×10^{-7}

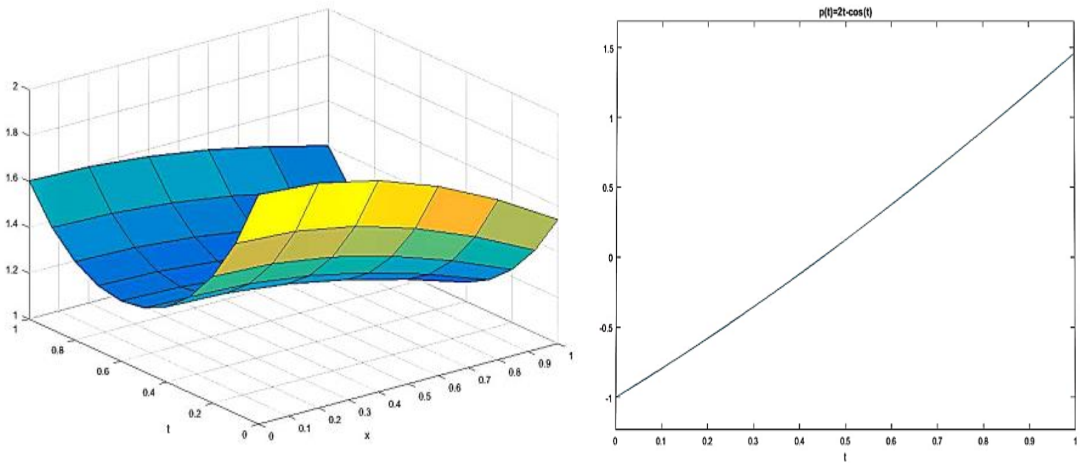


Fig. 6. The space-time graph of exact solution u(x,t) (left) and time graph of p(t) (right).

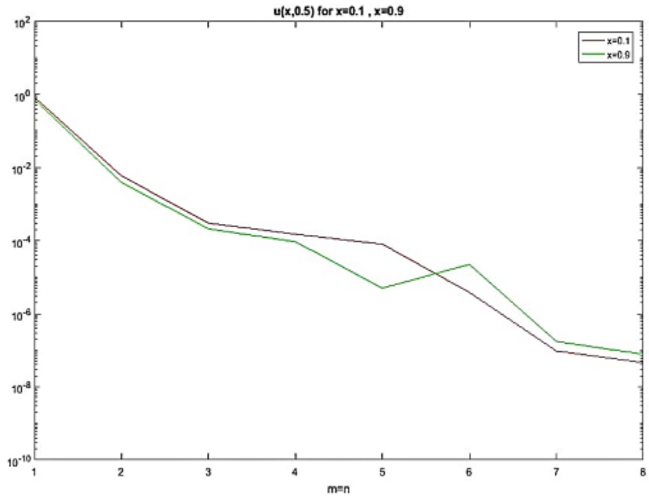


Fig. 7. Graph of the absolute error u(x, 0.5) for x=0.1 and x=0.9 with various value of m=n.

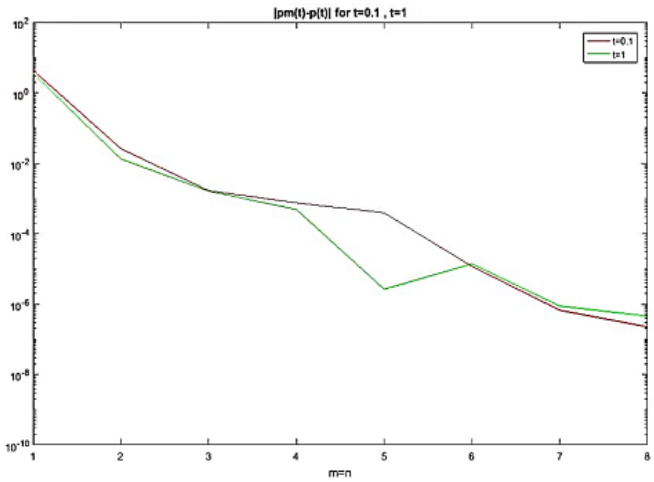


Fig. 8. Plot of $|p_m(t) - p(t)|$ for t=0.1 and t=1 with various value of m.

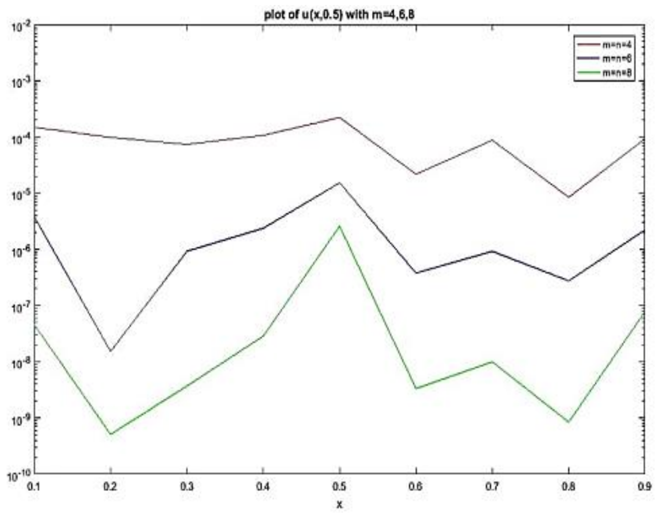


Fig. 9. Graph of the absolute error for u(x, 0.5) with m=n=4, 6, 8.

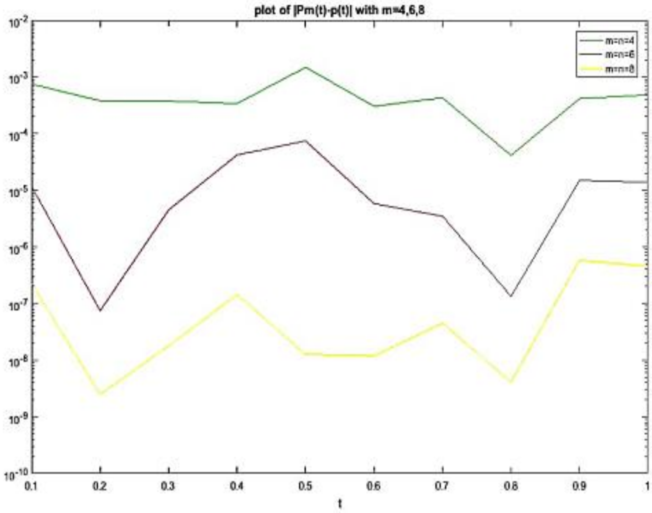


Fig. 10. Plot of $|p_m(t) - p(t)|$ with m=4, 6, 8.

The obtained results from *Tables 1* to *4* showed that this approach can solve the problem effectively. The described computational method produces very accurate results even when employing a small number of collocation points. And also, *Figs. 2* to *5* and *Figs. 7* to *10* show the reduction in the error for the function u(x,t) and control parameter p(t) by increasing the value of m, n.

6 | Conclusion

In this study, the inverse problem for heat equation is discussed. The SCT method is presented to solve the equation. The numerical approach is to expand the unknown function and unknown control parameter in terms of the shifted chebyshev of the first kind and the tau method so that it reduces the problem into a system of algebraic equation. The obtained results showed that this approach can solve the problem effectively. The new described computational technique produces very accurate results even when a small number of collocation points are employed.

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The Impact of Internal and External Driving Forces and Strategic Decisions on Supply Chain Risk Management (Case Study: Automotive Industry)

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Abstract

The primary purpose of this research was to understand the importance of supply chain strategies in the field of supply chain risk management, emphasizing the effectiveness and efficiency of agile and lean strategies to create resilience and robustness in the supply chain. Data was collected from 392 supply chain experts working in Iran's automotive industry to test hypotheses through structural equation modeling. The findings of this study show that Market Orientation (MO) (as an external force) has more signicant impact on the development of Agile Strategy (AS) than Lean Strategy (LS). In contrast, the Quality Management (QM) system (as an internal force) is highly correlated with the development of lean supply chain strategies. Moreover, agile and lean strategies also have a signicant impact on a resilient and Robust Supply Chain (RB). The proposed model helps organizations understand and create an ideal supply chain by implementing the right combination of both agile and lean supply chain strategies, which in turn helps to create a resilient and RB. Therefore, the findings of this study help policymakers to improve supply chain strategies by incorporating new management practices. This is original research that has various valuable insights for academic researchers and also supply chain strategy professionals as it reveals empirical evidence of the past vital concepts.

Keywords: Quality management, Market orientation, Risk management, Agile strategy, Lean strategy, Leagile strategy, Supply chain resilience, Supply chain robustness.

1 | Introduction

A supply chain includes all the steps and parts that directly or indirectly affect the supply of customer demand [1]. Therefore, the supply chain includes not only the manufacturer and suppliers but also warehouses, retailers, transportation and customers [2]. Focusing on improving supply chain performance is one of the critical elements required to meet customer needs and gain a sustainable competitive advantage, and therefore is of great interest to managers and researchers [3]. Businesses are highly dependent on their supply chain for their success and sustainability. Every business consists of one or more parts of the entire supply chain. Therefore, paying attention to the supply chain of any business is one of the essential priorities of senior managers. Trying to effectively manage this chain is the key to victory [4].

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Due to the scarcity of resources and the need to conserve resources for the next generation, significant growth in research and development has been done to create a sustainable market-focused production system to reduce supply chain waste and improve product recovery. On the other hand, create a stable, rapid reaction production system to produce a various of products required by customers [5]. Despite the increasing attention of managers, the impact and frequency of disruption in the supply chain is very high. Whenever there is a disruption in any network node, it causes a bottleneck in the supply chain and stops the entire supply chain network. Disruption can take many forms, including delays in order, quality problems in production, breakdowns of machinery, as well as natural disasters and catastrophic events, and its effects can vary from the operational level to the strategic level. According to Sodhi et al. [6] risk in the supply chain network harms supply chain performance in terms of efficiency and responsiveness. So, supply chain managers tend to be equipped with strategies for dealing with and managing risk. Ivanov et al. [7] confirm that resilience and robustness are two essential capabilities to manage supply chain risk effectively. This point of view is confirmed by Wieland and Wallenburg [8], who believe that in response to any disruption, the supply chain should be resilient and robust. Robustness is the operational capability that enables the supply chain to endure rather than adapt to disruptions. Resilience, on the other hand, is an active and responsive operational ability to deal with major and minor disruptions [8].

Effective risk management has always been one of the characteristics of successful firms to maintain competition in the long run. Because of the risky business environment, it can be tough for even senior supply chain planners to predict the outcome of their plans, decisions, and strategies. So, according to Deloitte's theory, strategic risks are one of the important reasons that can negatively affect supply chain risk management capabilities. Strategic risks influence or are shaped by strategic business decisions. Limited studies have focused on selecting and applying appropriate supply chain strategies to reduce risk. In addition, supply chain risk management and supply chain strategies are highly interrelated. Carvalho et al. [9] believe that the supply chain suffers costly disruptions. Therefore, senior managers must develop skills to reduce its adverse effects. It can be inferred that with proper implementation of supply chain strategies, its vulnerabilities will eventually be reduced [9].

Agile and lean supply chains are two strategies that any organization can choose to control its operations. Both strategies are often interrelated. Many scholars have argued that implementing a Lean Strategy (LS), due to its focus on minimum inventory and intensive planning, alone is not appropriate for the supply chain. Even agile implementation may not be cost-effective for firms. Thus, Naylor et al. [10] was the first researcher to introduce the concept of integrating both strategies into one supply chain, namely the leagile supply chain strategy. By using the leagile strategy, one can reap the benefits of both strategies [10]. Ambe states that the implementation of both agile and lean strategies enables companies to reduce costs, improve quality and respond to customer demand, while maintaining sustainability of the company's supply chain. Supply chain risk can be controlled by combining two agile and lean strategies [9]. The results of Rahimi and Alemtabriz [4] research in the Iranian military industry showed that Agile and lean strategies are intertwined, and their simultaneous implementation leads to improved supply chain performance [4]. There is limited empirical research in supply chain risk management and supply chain strategies such as lean and agile. Given the importance of two Agile and lean paradigms in improving the economical and rapid response of the supply chain to customer needs, implementing these paradigms is essential for the supply chain of products. Managers do not know which of these paradigms is a priority. Without this knowledge, not only will they fail to apply these paradigms properly, but they will also waste significant financial resources. The current research is designed to present a model of the leagile hybrid paradigm to answer these questions and resolve the ambiguity of managers in applying the leagile hybrid paradigm in the supply chain. On the other hand, from the contingency theory, theorists have argued that the efficiency, effectiveness or appropriateness of risk reduction strategies depends on the internal and external environment of the organization. Therefore, to solve a problem, there is no specific strategy consistently responsive in all situations.

Therefore, this research pursues several goals. The first step is determining how agile and LS decisions affect risk management ability. The second step is to try to discover and explain the driving factors that help to formulate the appropriate combination of such strategies. Finally, the purpose of this research is to propose a framework that can help companies to create a robust and resilient supply chain with the help of these strategies. This study is not only contributing by bridging the gap by examining the impact of agile and lean strategies on supply chain resilience and robustness, but also focuses on how to improve supply chain risk management by balancing agile and lean strategies.

This study provides academic assistance and various strategic implications for managers and researchers. This paper is structured into five sections. Section 2 reviews theoretical foundations and the previous literature. In Section 3, the research method is explained. Section 4 presents the research findings using statistical analysis and finally, in Section 5, the conclusions and consequences of this research are discussed.

2 | Theoretical Background and Hypotheses

In this section, a theoretical and empirical review of the literature is presented. It has six sections. Section 2.1 provides an in-depth review of the most essential resource-based perspective theory. Supply chain risk management is explained in detail in Section 2.2, which includes two critical, strategies, namely resilience and robustness. Section 2.3, it provides theoretical foundations related to supply chain strategies, ie lean, Agile Strategy (AS), and a combination of both supply chain strategies. Section 2.4 discusses the internal and external forces needed to guide agile and lean strategies. In section 2.5, a summary of the background of the research is presented, and Section 2.6 explains the research hypotheses.

2.1 | Resource-Based View

The Resource-Based View (RBV) is a new approach to the theoretical issues of strategic management, which considers the organization's core resources, capabilities, and competencies as the basis for formulating strategies. In this view, the effectiveness of resources in driving the organization towards success is not the same. In the framework of the value chain, resources are transformed into capabilities, then into core competencies, and finally into competitive advantages. Therefore, in the process of strategic planning, the fundamental issue is to identify strategic capabilities, as well as to operationalize and measure them [11].

RBV focuses on the internal strengths and weaknesses of the organization, not on the opportunities and threats that arise from outside the organization. Firms need to focus on improving business performance by developing a resource-focused strategy and becoming a force for competitive advantage [12]. There is a belief that resources belonging to the organization are the main element of organizational competitiveness [13]. The primary function of the strategy is to create a competitive advantage, and the strategy should turn the organization's resources into a competitive advantage. This transformation occurs through the chain of resources, ability, core competency and competitive advantage. This issue is shown in *Fig. 1* [14].

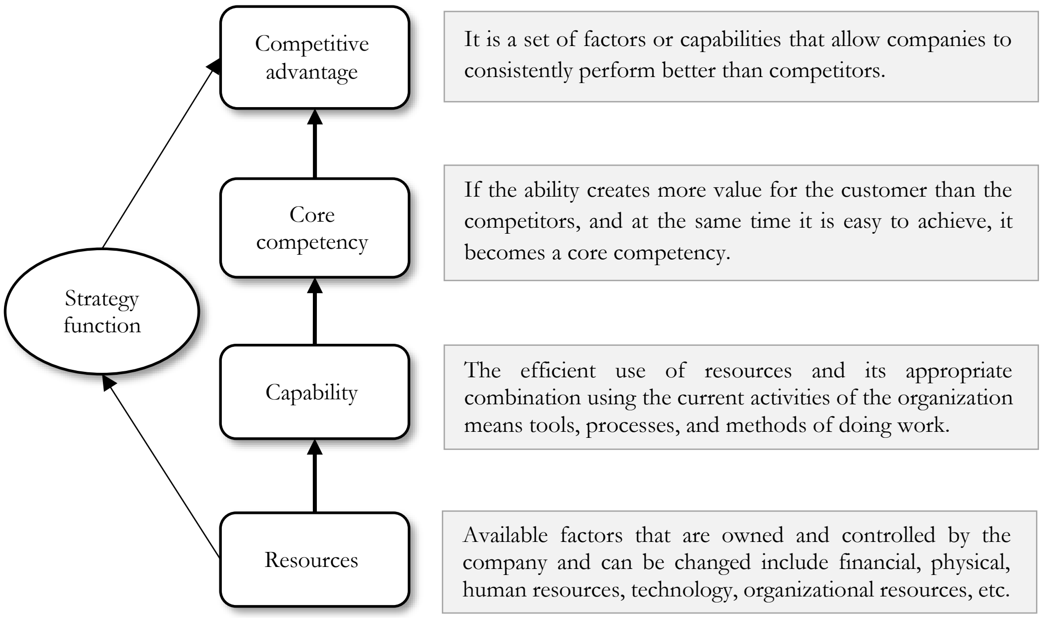

Fig. 1. RBV and the function of strategy.

RBV is a theoretical approach that emerged as a response to the disruption in the business environment caused by globalization, economic and political crises, and technological innovations. Based on RBV theory, organizations can achieve a competitive advantage by having valuable, unimaginable and irreplaceable resources. Resources can be classified into the following categories: human capital resources, physical capital resources, and organizational capital resources. The coordination of all these resources can improve the performance, competitiveness and sustainability of firms. Firms must build a set of strategic resources and capabilities to achieve sustainable competitive advantage by seizing opportunities and reducing risk. In this regard, resources must be aligned to shape capabilities following strategic organizational requirements in order to create or protect value [15]. According to the RBV theory, risk management capabilities consist of two parts: supply chain resilience and robustness. Since supply chain disruptions can have severe and long-term economic effects, resilience and robustness may be developed at the strategic level to deal effectively with supply chain disruptions. Developing supply chain resilience and robustness has tangible consequences for risk management. Thus, RBV theory is the basis of the primary theoretical model to discuss supply chain resilience and robustness [9].

2.2 | Supply Chain Risk Management

Supply chain risk management is a crucial management challenge that affects the organization's performance. All economic disruptions, natural and unnatural, and unforeseen threats affect the performance and profitability of supply networks. In other words, supply chain disruptions are unplanned events that may occur in the supply chain and affect the normal production flow. By increasing the risk of the supply chain, events that lead to disruptions in the flow of materials can cause large-scale disruptions. These disruptions may spread throughout the supply chain, and if supply chain activities cannot manage well the unforeseen disruptions, they will face potential negative consequences, and this will increase the business continuity risk and cause financial losses. Therefore, one of the challenges of today's business is to manage and reduce risk so that the supply chain is not damaged. Due to being in the sensitive region of the Middle East, the threat of natural disasters such as floods and earthquakes, and unique political and economic conditions, our country is highly exposed to all kinds of risks and, naturally, disruptions in all kinds of supply chains. Supply chain Quality Management (QM) measures, as one of the critical components of supply chain management and the development of QM programs, include not only traditional measures within the organization, but also include external measures across the organizational boundaries, which creates the integration of the company with its suppliers and customers [16].

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To reduce vulnerabilities and ensure continuity throughout the supply chain, supply chain risk management can be used. It broadens the traditional risk management perspective by linking upstream and downstream supply chain risk. In summary, supply chain risk management identifies, evaluates, and controls uncertainty and potential risk in the supply chain, and reduces disturbances in both reactive and proactive ways. As mentioned earlier, the two main strategies of supply chain risk management, namely robustness which is proactive and resilience which is reactive nature in supply chain risk management, are important capabilities for effective supply chain risk management [8].

Robustness can be defined in terms of supply chain risk management as "the ability of the supply chain to withstand and manage its performance in the face of internal and external turbulences" [17]. This definition focuses on the ability to continue operations while resisting the effects of supply chain turbulences by providing alternative resources or, if an emergency plan is needed, for its rapid implementation. Robustness is considered a proactive strategy to cope with environmental changes, disruptions, or turbulences. Tang [15] defines a "robust strategy" as a strategy that will enable an organization to efficiently manage regular fluctuations under normal circumstances regardless of the occurrence of significant disruptions and helps an organization to sustain its operations during the disruption [15].

Resilience first appeared in materials science to describe the physical ability to return to its original state after any deformation. It is now emerging as a theory. The term supply chain resilience refers to the ability of the supply chain to restore its regular operation after realizing the effects of risks, threats and vulnerabilities [18]. Resilience by allowing businesses to develop goods with features and performance that Satisfies the needs of their customers, improves customer satisfaction [12]. One of the required features of any supply chain to continue operating and remain competitive, not only to resolve disruptions in the short term, but also to create the ability to adapt to changes and improve the organization in the long term. Management strategists argue that supply chain resilience is the ultimate competitive advantage in the present age. However, the literature has mainly focused on supply chain characteristics to determine the level of risk or resilience to external (such as floods, terrorist attacks, earthquakes, etc.) and internal (such as failure of actors in the supply chain) disturbances. Sheffi and Rice [19] define supply chain resilience as a state in which any firm can quickly recover and return to its ideal state after the disruption. In addition, Ponomarov and Holcomb [20] described resilience more effectively, stating that resilient firms can respond to disruptions at the desired level with management, control and improve operations in the supply chain networks through contingency and adaptability. Resilience is important because of the problems associated with risk management, as it enables the firm to be prepared to deal with risks posed by internal processes, suppliers and customers [9]. The main distinction between robustness and resilience is that resilience refers to the power of the supply chain to deal with disruptions and maintain operational processes as planned. In contrast, resilience describes the ability to recover to the initial state after the effects of disruptions have been internalized [18].

2.3 | Supply Chain Strategies

Lean supply chain strategy is a cost-based approach that proactively improves supply chain performance by reducing or eliminating all non added value activities at all stages of the product life cycle, from product design to final delivery to the customer [19]. The implementation of a LS is based on the fact that supplier organization is selected on the basis of quality and cost, in addition to the full use of capacity; Scale savings and technology optimization are considered to ensure the most efficient and accurate data transfer; The link between information must be developed Implementing this strategy will reduce costs, increase inventory turnover and prevent wastage in processes. Such benefits increase the desire of firms to implement LS in their supply chain [9]. In the 21st century, the production level of the company depends on customer demand for customer satisfaction. Manufacturing companies are constantly not only improving their services, processes and products, but also thinking of introducing new products to the market to expand their relationship with customers. An industry that needs high volume and high quality production with lower production cost and high efficiency compared to them, lean production is the best offer. This has led to complex production planning as well as control system that has made mass production of goods difficult especially in the automotive industry and the manufacturing sector that is struggling with competitive markets. Many automotive and manufacturing industries have switched to lean production because lean production meets consumer demand by reducing waste. The primary goal of lean industrial production is to provide products with the lowest cost (reduction of waste) and the shortest time, in order to satisfy consumers. Various researchers focus on their research paper benefits of lean implementation such as cycle time, inventory, waste, failure, Overall Equipment Effectiveness (OEE) set up time etc. [21].

In an era when industries are changing the business environment, every organization faces challenges, so to improve the supply chain, an AS must be used [21]. The agile approach is based on the fact that in the current turbulent environment, organizations must offer new, high-quality products to the market in the shortest time in order to retain current customers and attract new customers [21]. Agile supply chain strategy can be considered as benefiting from resilience and adaptability in order to respond dynamically, quickly and continuously to the changing needs of customers and the competitive environment. By implementing this strategy, the firm effectively adapts to the market, responds more quickly to customer demand, and cooperates more effectively with suppliers. Customer responsiveness is critical at all levels of the supply chain, because it is a competitive market need that organizations must understand customer demand and anticipate market changes. Dubey and Gunaskaran [1] have stated that customer responsiveness is an influential factor in globalization and can be achieved by adopting an AS [9].

Applying a lean supply chain in an organization leads to improvements in the form of cost reduction, high inventory turnover, defect prevention and shorter delivery time. These benefits have forced organizations to upgrade their supply chain based on lean principles. Although the lean supply chain has reduced production costs, this supply chain model has failed to be flexible to customer demand. This has caused managers to have the desire to use and develop the agile supply chain model to be considered as an alternative to the lean supply chain. Nevertheless, the concept of lean remains a prerequisite for an agile supply chain. The need for organizations to respond quickly and strongly to fluctuations in demand, in the form of volume and variety, has motivated them to move from lean strategies to agile strategies [23]. Findings indicate that there are a set of criteria specific to LS, such as process-focused criteria, cost, productivity, inventory, and delivery-based criteria, and specific criteria for agility, such as flexibility, responsiveness, information sharing and collaboration. Also, agile and lean strategies have commonalities in terms of quality, time and customer satisfaction. Lean practices are objective and focus on the organization's internal processes, while agile practices target the external environment [24]. A comparison of the main features of two agile and lean strategies is presented in *Table 1* [23].

Table 1. Key characteristics of agile and lean strategies.

Characteristic	Lean Supply Chain	Agile Supply Chain
Product type	Widely used products	Innovative products
Variety of products	Few	A lot
Product life cycle	Long	Short
Market demand	Predictable	Variable-unstable
Attractiveness for customers	Lower price	Faster access
Total marginal profit	Down	Top
The main cost in the system	Physical costs	Buying and selling costs
Penalty for shortage of goods in the system	Long-term contracts	Immediate and variable
Chain purchase policy	Buying goods	Allocation of capacity
System information capability	Optimal	Mandatory
System prediction mechanism	Algorithmic	Consultative

In recent years, another business philosophy called Leagile, a combination of agile and lean thinking, has emerged as one of the choices for the supply chain of organizations, so that both the benefits of being lean (eliminating types of waste) and flexibility it has agility. This approach is generally related to products that are assembled according to the customer's order. Because its demand forecast has very low volatility and relatively high accuracy [23]. Leagile supply chain strategy is a combination of agile and lean strategies, named by Naylor et al. [25]. Companies today tend to be flexible and responsive in a cost-effective manner.

Overall, this hybrid strategy provides the advantages of both agile and lean philosophies for the organization. Leagile supply chain strategy provides cost-effective resource control, flexibility and operational adaptability upstream and downstream of the supply chain in an unpredictable market environment [9]. The results of research by Srinivas et al. [26] show that inadequate human resource management, insufficient technology and innovation, and financial constraints are among the barriers to leagile strategy [26]. Yadav and Kumar [27] confirmed the positive effect of using leagile strategy in improving the operational, economic and even environmental efficiency of the supply chain [27].

2.4 | External and Internal Forces

A review of the past literature shows two influential driving forces of strategic management. A Market Orientation (MO) is an external factor that allows any firm to cope with changing market conditions by offering a high quality product. On the other hand, the QM system focuses on continuous improvement of the supply chain, which provides superiority in internal and controllable operations. So, it can be said that with MO, external opportunities and threats are fully understood and then for planning and proper implementation of strategies, through QM system, the use of available resources is closely monitored, which will bring the desired result [28].

MO has attracted the attention of researchers for more than two decades. Marketing in a company is said to promote the concept of the market. The philosophical background of marketism is the concept of marketing. Cultural MO is an organization that, with maximum efficiency and effectiveness, provides the necessary behaviors to create superior value for customers and, as a result, continuous superior performance for business [29]. Market-oriented companies must analyze the strengths and weaknesses of competitors and, on the other hand, use the knowledge to develop and implement strategies to create sustainable advantage and superior performance [30]. MO enables firms to understand the needs and expectations of their current and future customers. This market intelligence is then shared across all departments of firms to improve operations. According to Day [31], MO connects the organization to the external environment by focusing on the changing needs of the customer, the movement of competitors, environmental uncertainty and shareholder expectations. In addition, Ahire et al. [32] also acknowledged that organizations need to be market-oriented in response to unpredictable market conditions that allow them to gather market information so that they can adapt to it. Tseng and Liao [33] also stated that market-oriented organizations by collecting, disseminating and responding to market information with the help of inter-departmental coordination, enable the organization to perform well with diverse customer demand and unbridled competition in the market [9]. According to the opinion of Kohli and Jaworski [34], successful MO has three main requirements, which are: customer focus, coordinated marketing, and inter-department coordination in the firm. The firm must understand customer needs, so that firm is able to apply orientation, either reactive or proactive orientation. Reactive orientation usually makes firms focus on understanding and satisfying the current needs of the consumer [30]. The above discussion leads to that MO act as sensor which drives organizations to shape their strategies accordingly.

QM system as an essential strategic management tool has been considered in recent decades to create strategies in practice. Quality practices and their applications are involved at all levels of organizations to optimize resources. QM system is a concept related to the management procurement of resources until the delivery of the product to the final destination and even after-sales service. In addition, QM system is always looking for continuous improvement to achieve sustainability, competitiveness and efficiency. Talib et al. [35] showed that Implementing and improving the quality culture in the organization can increase organizational performance through engaging employees, improving relationships with suppliers and employees and customer orientation. Top managers who seek quality and are actively involved in QM, in fact, strengthen and change the strategies, vision and value of QM in the organization. Thus, QM system must be done extensively through management from top to bottom levels and all operational processes of the organization. The results of Wang et al. [36] research showed that quality improvement throughout supply chain processes leads to cost reduction, improved resource utilization, and improved process efficiency. In an organization, excellence can be achieved through the proper implementation of a QM system [37].

2.5 | Research Background

In the following, a summary of previous research is presented in *Table 2*.

Table 2. Summary of previous research.

Author(s)	Research Title	Result
Mohr-Jackson [38]	Conceptualizing total quality orientation	The author synthesizes existing knowledge on the topic and provides a foundation for future research by outlining the scope of the total quality orientation construct and providing an operational definition.
Lai [39]	MO in quality-oriented organizations and its impact on their performance	The findings of this research show that MO is positively and strongly related to the application of QM and business performance.
Min et al. [40]	A MO in supply chain management	MO is the basis for supply chain management and supports the implementation of QM.
Dehghan KhalilAbad and Aref [41]	Investigating the impact of supply chain QM practices and capabilities on operational and innovation performance (a case study of food industry companies in Mashhad)	Today, the competition from the company level has been moved to the competition between their supply chains, and an agile and lean supply chain is considered as a very important and decisive competitive advantage in the field of competition. The results show that the capabilities and actions of supply chain QM have an impact on innovation and operational performance.
Karim and Zaman [42]	A methodology for effective implementation of lean strategies and its performance evaluation in manufacturing organizations	Market oriented organizations execute the LS well because both are used to meet customer expectations.
Ghanbari et al. [43]	The impact of strategic cost management on the relationship between supply chain practices, top management support and financial performance improvement	Strategic cost management has a positive and significant effect on the relationship between different measures of non-financial performance of the supply chain and different measures of financial performance improvement.
Zelbst et al. [44]	Relationships among MO, JIT, TQM, and agility	Market oriented directly and positively impacts AS. Updating external information and maintaining constant communication with customers, competitors and shareholders allows the manufacturing company to quickly respond to changes in the volatile market.
Wang and Wei [28]	The importance of MO, learning orientation, and quality orientation capabilities in TQM: an example from taiwanese software industry	The importance of QM is not only limited to providing high quality products or customer satisfaction, but also plays an essential role in reducing supply chain costs in an efficient way by continuously improving process performance.
Nicholas [45]	Hoshin kanri and critical success factors in QM and lean production	QM and lean production is a solution for focusing the organization, aligning goals and plans at all levels, integrating goals and strategies.

Table 2. Continued.

Author(s)	Research Title	Result
Krehbiel and Miller [46]	Should agile be part of your QM?	AS is not so comprehensive and complete that it can be considered as an independent QM. However, it can be an important part of a QM approach.
Imam Vardi Malik et al. [23]	Leagile supply chain evaluation model for apparel	The assessment of Leagile of the supply chain helps organizations to identify and prioritize the indicators and criteria of the leagile of the supply chain, choose the appropriate supply chain approach from among the proposed approaches, and the specified indicators and criteria of leagile according to Prioritize them among the components and elements of the organization's supply chain.
Junaid et al. [47]	A neutrosophic ahp and topsis framework for supply chain risk assessment in automotive industry of Pakistan	The three criteria of agility, resilience and robustness are considered as the main coping strategies for risk management.
Christopher and Rutherford [48]	Creating supply chain resilience through agile six sigma	A supply chain robustness results from a LS. While supply chain resilience is related to AS.
Blackhurst et al. [49]	An empirically derived framework derived framework of global supply resiliency	An agile supply chain has resilience and due to this ability it can adapt itself to changes in the environment and recover quickly after a disruption, which has a positive effect on the performance of the chain.
Derakhshi Khajeh and Jabarzadeh [16]	Developing a causal model of factors influencing supply chain resilience	Based on field studies, the main effective components for increasing resilience and reducing recovery time have been identified, among which the importance of risk management in resilience through reengineering, agility, integration and flexibility of the supply chain. It is strengthened by investing in the development of growth and learning of the organization.
Amir taheri et al. [50]	The impact of supply chain strategies on supply chain integration and competitive performance (case study: Kerman tire industry)	The results of this research show that a lean supply chain is suitable for companies with higher priorities for quality, cost and delivery strategies. On the other hand, an agile supply chain is suitable for companies that compete in a flexible strategy
Wieland and Marcus Wallenburg [51]	Dealing with supply chain risks: linking risk management practices and strategies to performance	Having a Robust Supply Chain (RB) in the organization can help to achieve a resilient supply chain.
Ahmed and Huma [9]	Impact of agile and lean strategies on supply chain risk management	MO as an external force has a greater influence on the application of AS compared to LS, on the other hand, QM as an internal force has a greater influence on the development of lean supply chain strategies. In addition, lean strategies have a significant impact on a RB and agile strategies have a significant impact on a resilient supply chain. A RB can also help achieve a resilient supply chain.
Safavi Mirmahalleh et al. [52]	A model for risk management in the supply chain of Iran's gas industry	The results showed that external risks, production and transmission risks, organizational risks are important in risk management in the country's gas industry supply chain. Also, production, maintenance, transferand transmission and external risks are the highest risk in the first priority of control by the country's gas industry management.

Table 2. Continued.

Author(s)	Research Title	Result
Goli and Kianfar [53]	Designing a mask closed loop supply chain network by mathematical modeling and fuzzy multi-objective optimization approach	The goals of maximizing the final profit and minimizing the environmental impact are in conflict with each other because reducing the environmental impact leads to an increase in the total cost and reduces the profit of the organization. Also, the increase in demand for various types of masks increases the profit of the chain linearly, but its effect on the environmental effects of the chain has a completely non-linear behavior.
Mohammadi et al. [54]	The effect of production and strategic resilient and supply chain agility on performance (case study: Iran Khodro company)	Strategic resilient has a positive effect on company performance. The agility of the supply chain has an impact on the company's performance. Supply chain agility mediates the effect of strategic resilient on company performance.
Mohammad and Kazemipoor [55]	An integrated multi-objective mathematical model to select suppliers in green supply chains.	An integrated approach for supplier selection in the supply chain and order policy from each of them was investigated in this study. In order to achieve the goals of the research both multicriteria techniques, to select the suppliers (a strategic decision), and optimization methods, determine the optimal order level from each supplier and optimal routing (an operational decision) have been applied.
Rabbani et al. [56]	Presenting a new approach toward locating optimal decoupling point in supply chains	Locating supply chain decoupling point as a strategic decision was addressed in this paper. To do so, an AHP-DEA method was developed and the results were analyzed in a company from food industries in order to validate the proposed structure.
Chansamut [57]	Information system model for educational management in supply chain for thai higher education institutions	The research result shows that information system model for educational management in supply chain for Thai higher education institutions is consisted of 7 key elements which are 1 main element, 2 raw materials, 3 suppliers, 4 manufacturers, 5 service provider, 6 finished product 7 customers. The results from experts' agreement information system model for educational management in supply chain for Thai higher education institutions was a high level. It showed that information system model for educational management in supply chain for Thai higher education institutions could be used to develop information system.

According to the mentioned studies, this research examines the impact of agile and lean strategies on the robust and resilience of the supply chain. Also, the role of internal and external factors affecting these strategies in the automotive industry is examined. This study tries to bridge the gap in previous research by examining the impact of agile and lean strategies on the supply chain, on the other hand, by balancing agile and lean strategies, it also focuses on how to improve supply chain risk management.

2.6 | Hypotheses Development

According to the research background and theoretical foundations and to examine the relationship between driving forces, strategic decisions and supply chain risk management in the form of a comprehensive test, the following hypotheses have been formulated.

Hypothesis 1: There is a positive and significant relationship between MO and QM.

Hypothesis 2: There is a positive and significant relationship between MO and LS.

Hypothesis 3: There is a positive and significant relationship between MO and AS.

Hypothesis 4: There is a positive and significant relationship between QM and LS.

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Hypothesis 5: There is a positive and significant relationship between QM and AS.

Hypothesis 6: There is a positive and significant relationship between LS and supply chain robustness.

Hypothesis 7: There is no significant relationship between AS and supply chain robustness.

Hypothesis 8: There is no significant relationship between LS and supply chain resilience.

Hypothesis 9: There is a positive and significant relationship between AS and supply chain resilience.

Hypothesis 10: There is a positive and significant relationship between RB and resilience supply chain (LR).

Based on the hypotheses presented above, the conceptual model of this study is shown in *Fig. 2*.

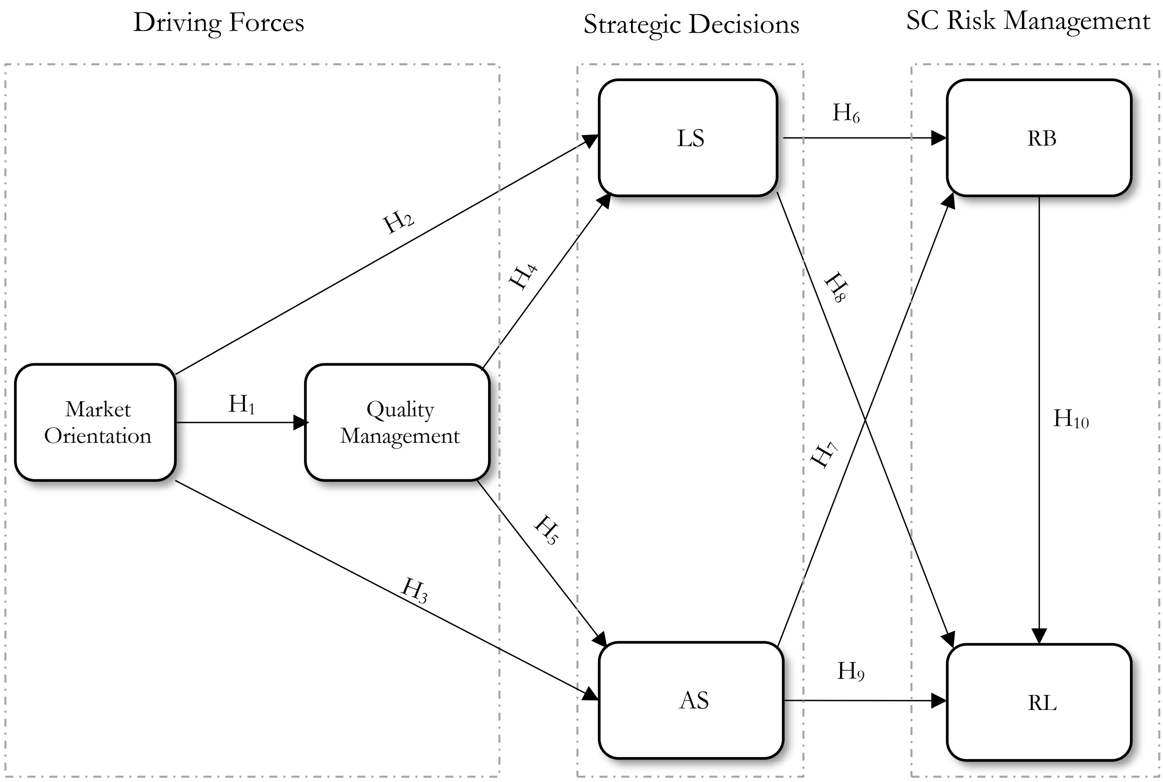

Fig. 2. Conceptual model of research.

3 | Research Methodology

The present research is applied in terms of orientation, case study in terms of strategy, descriptive and survey in terms of purpose, and tries to generalize the results to the statistical population by using data collected from samples. The statistical population of this study includes managers and experts in the supply chain of automobile firms [58]. Due to the size of the firms under study, the size of the population is unknown and based on the Cochran's formula (at a 95% confidence level) the minimum sample size will be 384 people. The structured questionnaire was distributed through face-to-face and electronic referrals. For electronic cases, follow-up messages were often sent to remind key respondents as well. A total, 392 valid responses were received. This questionnaire has 22 questions with 5-point Likert scale, which are evaluated from 1 to 5 (where 1 = strongly disagree and 5 = strongly agree). The variables have been adapted from [9]. In this study, Structural Equation Modelling (SEM) technique and Confirmatory Factor Analysis were used, and SPSS and AMOS software were used to model structural equations. The flow process of this applied research is shown in *Fig. 3*.

Fig. 3. Flow chart of research process.

4 | Research Findings

4.1 | Descriptive and Demographic Statistics of the Research

The demographic characteristics of this study are presented in *Table 3*.

Table 3. Sample demographics summary.

Demographics	Classification	Frequency	%
	Man	316	80.6
	Female	76	19.4
	Less than 30 years	34	8.8
	31 to 40 years	68	17.3
	41 to 50 years	162	41.3
	More than 50 years	128	32.6
	Diploma and lower	38	9.7
	Associate and bachelor	248	63.2
	Masters	82	21
	Doctoral	24	6.1
	Less than 5 years	53	13.5
	5 to 10 years	167	42.6
	11 to 20 years	97	24.7
	More than 21 years	75	19.2
Total		392	100

According to the table, about 81% of the participants are men. People aged 41 to 50 years (with 41.3%) were among the most participants, and people aged less than 30 years (with 8.8%) were among the lowest participants. People with an associate's degree and a bachelor's degree (with 63%) are the most respondents. This means that most participants have at least a university degree. The work experience of most respondents, with a frequency of 167 people, is between 5 and 10 years. Information about descriptive statistics of variables is shown in *Table 4*.

Table 4. Describes the research variables.

Construct	Average	Min	Max	Sd
MO	3.27	1.33	5.00	0.78
QM	3.41	1.67	5.00	0.64
LS	3.33	1.33	5.00	0.66
AS	3.48	1.33	5.00	0.63
RB	3.24	1.25	4.50	0.64
RL	3.23	1.50	4.75	0.69

As can be seen, the highest average of respondents for the AS component was 3.48, and the lowest average of respondents was related to the RL component of 3.23. Also, the highest standard deviation is related to the MO variable (0.78), and the lowest standard deviation is related to the AS variable (0.63).

4.2 | Validity and Reliability of Research Tools

Before testing the hypotheses, the validity and reliability of the research variables should be checked using the confirmatory factor analysis test. For convergent validity test using confirmatory factor analysis, the factor loads of each variable must be greater than 0.4 and significant. On the other hand, Average Variance Extracted (AVE) should be greater than 0.4. The results showed that the factor load of all items is more than 0.5, which for the sake of brevity, the tables were not presented. The results of AVE are presented in *Table 5*, so the convergent validity of the questionnaire is confirmed.

Table 5. AVE results for convergent validity.

Construct	MO	QM	AS	LS	RB	RL
AVE	0.762	0.510	0.661	0.546	0.605	0.683

To examine the discriminant validity of the variables, the Fornell-Larcker matrices were used and the results of which are presented in *Table 6*. The primary diameter of this matrix is the square root of the AVE values, and only the first-order hidden variables are entered in the matrix. According to the Fornell-Larker matrix, the value of AVE denominators is greater than the correlation value between them, which indicates good discriminant validity, and good fit for the measurement model.

Table 6. Fornell-Larker matr ix results for discriminant validity.

Construct	MO	QM	AS	LS	RB	RL
MO	**0.873**					
QM	0.518	**0.714**				
AS	0.409	0.619	**0.813**			
LS	0.425	0.687	0.524	**0.739**		
RB	0.512	0.336	0.445	0.632	**0.778**	
RL	0.476	0.510	0.648	0.421	0.360	**0.826**

Reliability determines the accuracy of the measurement. This means that if the research is repeated under the same conditions, the score or value of the questionnaire will not change much. To evaluate the reliability of the questionnaire, Cronbach's alpha test and compositional reliability were used, and their results are presented in *Table 7*. Since all coefficients are greater than 0.7, it shows the internal consistency of the results and the reliability of the questionnaire.

Table 7. Results of the questionnaire reliability.

Construct	Number of Questions	Cronbach's Alpha Coefficients	Compositional Reliability
MO	4	0.874	0.845
QM	3	0.795	0.709
AS	4	0.863	0.874
LS	3	0.801	0.843
RB	3	0.813	0.867
RL	5	0.903	0.729

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4.3 | Test the Normality of Data Distribution

Kolmogorov-Smirnov (KS) test was used to evaluate the normality of the distribution of research variables. According to *Table 8,* the hypothesis of normality of data distribution is accepted at the level of 5% error probability (hypothesis H_0 based on the normality of data distribution). In other words, the distribution of research data is normal.

Table 8. Results of data distribution normality test.

Construct	KS	Sig.	Result
AS	0.562	0.905	Normal
LS	0.647	0.796	Normal
MO	0.699	0.713	Normal
QM	0.984	0.715	Normal
RB	0.506	0.963	Normal
RL	0.974	0.299	Normal

4.4 | Predictive Relevance of the Model

In PLS, model fit is usually measured through R^2 and Q^2 (Stone-Geisser Criterion). The coefficients R^2 are related to the endogenous (dependent) variables of the model and show the effect of the exogenous variable on the endogenous variable. R^2 less than 0.30 is considered as low, between 0.30 and 0.60 is considered as moderate, and greater than 0.60 is considered as strong structural model fit. In this research, the amount of R^2 values as shown in *Table 9* indicate a moderate to strong model fit.

As mentioned above, Q^2 is another tool for measuring the capability of predictive relevance. This measure was produced in Smart PLS and is required to be equal to or greater than 0. A Q^2 value larger than zero for a particular endogenous latent variable indicates the PLS path model has predictive relevance for this construct. The test results are presented in *Table 9*.

Table 9. Structural model fit analysis.

Construct	R^2	Q^2
AS	0.46	0.293
LS	0.38	0.364
QM	0.48	0.158
RB	0.52	0.284
RL	0.67	0.421

4.5 | Hypotheses Testing

In this study, ten hypotheses were tested using the SEM technique. All hypotheses (except the seventh and eighth hypotheses) are statistically significant and verifiable. *Fig. 4* shows the path coefficients of the research hypotheses.

Details of the hypothesis test results are presented in *Table 10*. Regarding the hypotheses, the results indicate that MO has a positive and significant impact on QM ($\beta = 0.415$, $p < 0.001$). Therefore, the first hypothesis of the research is strongly supported. In this regard, MO has a positive and significant impact on LS ($\beta = 0.212$, $p < 0.001$) and AS ($\beta = 0.458$, $p < 0.001$). Therefore, the second and third hypotheses are also supported.

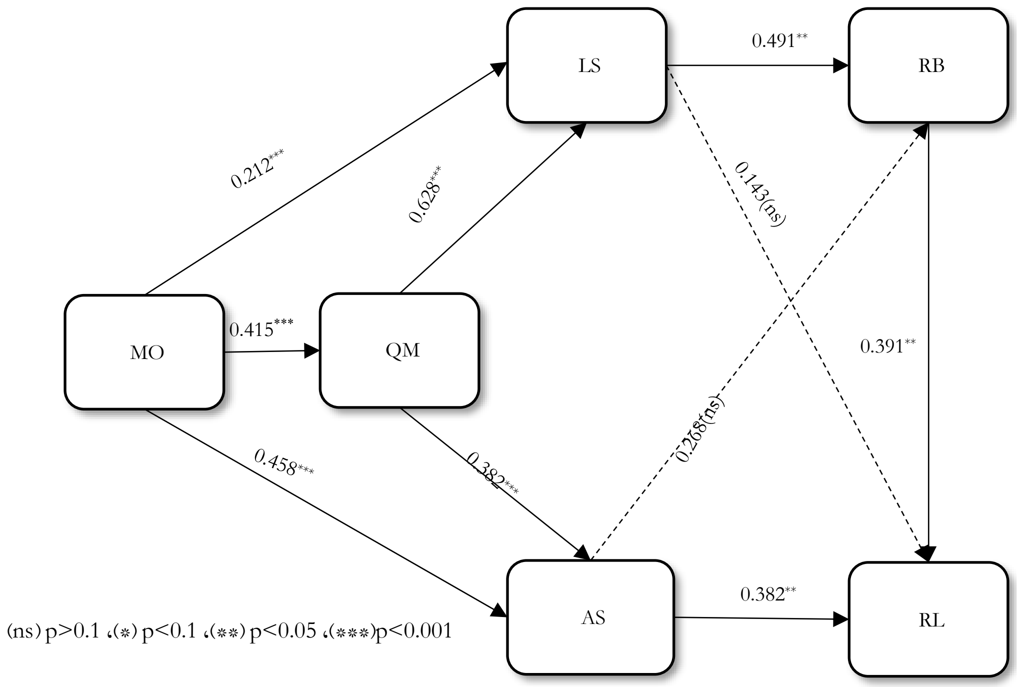

Fig. 4. Hypotheses testing.

QM has a positive and significant impact on two lean strategies ($\beta = 0.628$, $p < 0.001$) and AS ($\beta = 0.382$, $p < 0.001$). This result indicates that by increasing the level of QM, agile and lean strategies are applied in the supply chain better. Therefore, the fourth and fifth hypotheses of this research are also supported.

LS has a positive and significant impact on creating a RB ($\beta = 0.491$, $p < 0.01$), but LS does not have a significant effect on RL flexibility chain ($\beta = 0.143$, $p > 0.1$). According to the theoretical foundations of research, this relationship should be insignificant. Therefore, the sixth and eighth hypotheses are supported. The AS has a positive and significant impact on RL ($\beta = 0.382$, $p < 0.01$), but the LS does not have a significant impact on RB ($\beta = 0.268$, $p > 0.1$). According to the theoretical foundations of research, this relationship should be insignificant. Therefore, the seventh and ninth hypotheses are supported. Finally, creating a RB has a positive and significant impact on RL ($\beta = 0.391$, $p < 0.01$). Therefore, the tenth hypothesis is also supported.

Table 10. Hypotheses testing results.

Hypotheses	Relationships	B-Coefficient	T Statistics	P Values	Decision
H_1	MO→QM	0.415	11.581	0.0001	Supported
H_2	MO→LS	0.212	3.729	0.002	Supported
H_3	MO→AS	0.458	7.433	0.001	Supported
H_4	QM→LS	0.628	4.182	0.009	Supported
H_5	QM→AS	0.382	3.482	0.004	Supported
H_6	LS→RB	0.491	3.460	0.01	Supported
H_7	AS→RB	0.268	1.617	0.114	Supported
H_8	LS→RL	0.143	1.427	0.261	Supported
H_9	AS→RL	0.382	9.915	0.034	Supported
H_{10}	RB→RL	0.391	3.657	0.028	Supported

4.6 | Results of Research Model Fitting

After testing the research hypotheses, it must be ensured that the SEM has a proper fit. There is a wide range of fitness criteria and indicators that can be used to measure the overall pattern. *Table 11* shows the results of several indicators. As can be seen, all the fitting characteristics are within the proper range. Therefore, the desirability of the SEM in fitting the collected data is confirmed.

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Table 11. Research model fitting results.

Fitness Criteria	CIMIN/Df	GFI	IFI	TLI	CFI	NFI	RMSEA
The main pattern	4.501	0.990	0.951	0.990	0.948	0.930	0.009
Acceptable level	1 to 5	More than 0.9	More than 0.9	More than 0.9	More than 0.9	More than 0.9	Less than 0.05

5 | Conclusion and Recommendations

In such a competitive era where the changing market environment and the changing needs of customers have intensified, supply chain managers are more emphasizing on implementing dynamic strategies to get a quick response at the lowest cost. The aim of this research is to propose a conceptual framework for analyzing the drivers of leagile strategy and also to provide empirical evidence of the effectiveness of agile and lean supply chain strategies in creating a resilient and RB, and on the other hand, the impact of risk management on supporting agile and lean strategies. Initially, our results showed that QM is significantly affected by MO. This finding is consistent with the results of previous research. This means that the more market oriented and produce quality work, the more value it creates for the customer. MO is also significantly associated with both agile and lean strategies. Also, the results show that MO meets with customers and understands their needs, which will lead to a better direction of the strategies needed by the organization. Upon further investigation, the findings suggest that MO has a more significant effect on AS than LS. Lee [59] also supports this point of view, who suggested that an AS understands the changing needs of customers, therefore can respond effectively to the market. In contrast, LS can be helpful in environments where market demand is stable. Thus, it can be concluded that the AS can respond in times of uncertainty and is more market oriented than the LS. However, to drive both agile and lean strategies, supply chain managers must focus on the market and be aware of their customers' needs, as well as manage product quality accordingly to respond quickly to volatile markets by reducing non-value-added activities.

Based on the results of this research, QM as an internal driving force is effective in both agile and lean strategies. The application of QM makes it possible to maintain production quality, thus meeting or exceeding customer expectations. Previous studies have confirmed this hypothesis by stating that QM is a suitable tool for implementing lean supply chain in the organization. Manufacturers can gain a competitive advantage in an increasingly volatile market by providing quality products, fast and reliable delivery, and flexibility. So it can be concluded that the existence of QM is essential for the better implementation of LS compared to AS. Anyway, the findings emphasize that before implementing both agile and lean strategies, companies should focus on QM, as it directly helps drive both supply chain strategies.

The results of this study also show that lean strategies have a significant positive effect on RB. Currently, many benefits of implementing a LS for supply chain managers have been identified, the most important of which is the elimination of non-value-added activities that improves productivity, and simplifies operations, and ultimately makes supply chain robust. On the contrary, the present study concluded that the implementation of AS in the organization would not significantly lead to a supply chain robust. However, the evidence from this study shows that LS leads to robustness compared to AS. Therefore, to create a robustness supply chain, managers need to put more emphasis on implementing a LS.

This study supports the hypothesis that both agile and lean strategies are essential in supply chain risk management. Agility (unlike the lean supply chain) has a strong positive impact on resilience. The results of this research also support this concept, because the LS operates on the just-in-time technique, which means no buffer stock and less inventory, making unpredictable events challenging to deal with. According to the theoretical foundations and findings of this study, networks must be able to respond quickly to uncertain events in order to achieve a resilient supply chain. Therefore, manufacturing firm’s professionals must have the agility to create a resilient supply chain.

Also, the findings showed that a RB increases its resilience. The findings of this hypothesis are consistent with the results of previous research, as Wieland and Wallenburg [51] showed that AS and RB are two essential elements to creating a resilient supply chain. Therefore, it can be concluded that by implementing agile and lean strategies, organizations create robustness which leads to a resilient supply chain, and this is the ideal supply chain which that proactively and reactively controls its risk from upstream and downstream supply chains.

5.1 | Managerial Implication

This research considers QM as an internal driving force and MO as an external driving force to implement both agile and lean strategies. In addition, if there is no connection between the internal operations of the organization and its external environment, the customers' needs may not be properly and fully included in the production. The findings of this study emphasize that integrating MO and QM is critical to creating a resilient and RB in an unstable and dynamic environment. Most importantly, the managers are advised to focus first on the external environment and the customer, then transfer the customer's voice to QM, as QM has a greater impact on both agile and lean strategies.

This study also suggests that policymakers emphasize QM as the priority to provide quality products that should be commensurate with the customer's needs. In addition, the main goal of manufacturers is to reduce production costs and closely monitor the production flow, which reduces the possibility of defects. Therefore, the company's focus on QM (as an internal environment driving) will be necessary to implement both agile and lean strategies.

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The findings also suggest that automotive industry supply chain professionals should focus on implementing leagile strategy (the combination of agile and lean strategies) to create resilient and RBs, as this study shows that both agile and lean strategies work well when the voice of the customer is recognized, and information is shared within the organization with other departments. Both agile and lean strategies have advantages for the organization. The benefits of both strategies can be achieved by combining these two strategies. This proposed model helps supply chain professionals strengthen resilient supply chains by working to eliminate disruptions in the supply chain and improve responsiveness to customer, while improving robustness by cost-effectively implementing a LS. The main goal of implementing a LS is to reduce waste and make full use of resources. On the other hand, the main goal of the AS is to respond quickly and appropriately to unpredictable demands and dynamic environments. These results lead manufacturing organizations to how lean supply chains are critical to create robustness and how agile supply chains are critical, and the ability to create resilience which helps firms to determine the best direction for their ideal supply chain.

In the last few years, due to the intensification of international sanctions, automotive managers could not take an essential step in the field of raising the quality, and controlling the cost of products and creating diversity. Sanctions limited their access to quality raw materials and new technology, and the result was low-quality, expensive cars and unsatisfied consumers. Based on this, it is suggested that the domestic automobile manufacturing companies should not neglect the localization of production and the strengthening of the domestic supplier, as well as the activation of the research and development unit. A manufacturer with a LS should choose criteria to select a supplier, and a manufacturer with an AS should use criteria to support their agility.

5.2 | Future Research Recommendations

This study emphasizes the direct relationship between the factors affecting the supply chain, so in future studies it will be interesting to investigate some of the mediating effects. In addition, future studies can examine the effect of integration as a moderating factor. Also, examining the factors affecting QM and MO may improve the research model and create more perspectives. This study has been done in the automotive industry; due to the different environmental and operational conditions of industries, the implementation of this model in other industries is necessary.

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Internal Audit Program Planning and Implementation Determinants of an Automotive Company's ISO 9001 Quality System

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Abstract

This single case study examined how key internal audit planning and implementation determinants impacted a South African automotive company's ISO 9001 Quality Management System (QMS) objectives. The study used a mixed method approach; qualitative data nesting in quantitative data. The purposive sampling technique was used to collect primary data from managers, internal quality auditors, and auditees. Professional judgment was used to collect secondary data from the 2017 to 2020 audit reports. Descriptive data analysis was conducted on the data collected. The internal audits were conducted beyond departmental boundaries and organizational structures. The audit determinants were; compliance with the ISO 9001 standards, and maintenance of ISO 9001 QMS certification. The process and system internal quality audits were conducted to correct nonconformities before and after external audits. Audit reports from certification bodies also determined the scope of the subsequent internal audit programs for processes and systems. In addition, the internal auditors relied on their judgments and on the technical experts' advice to sample processes, areas, and material to be audited. management audit review reports also contributed to determining the scope of audit programs. Despite different stakeholders' contributions, the company's internal quality audit programs did not embrace customer focus and continuous improvement. The audit program was a reaction to internal and external stakeholders' complaints. However, the study is fundamental to improving the company's ISO 9001 QMS performance. It discusses issues that drive the planning and implementation of audit programs. The findings are likely to stimulate similar research in other sectors and on a bigger scale. There are also opportunities to evaluate the determinants related to monitoring, reviewing, and improving audit programs.

Keywords: Audit program determinants, Internal quality audit, ISO 9001 quality management system, Automotive company, South Africa.

1 | Introduction

The impact of internal quality auditing on the performance of ISO 9001 Quality Management System (QMS) depends on its effectiveness [1]. The ISO 19011:2018 standard defines effectiveness as the extent to which planned activities and results are realized [2]. Thus, to understand how internal quality auditing practices are planned and their results realized, an investigation of the practice of implementing and managing an internal quality auditing program is necessary [1], [3]. An audit program is an arrangement of a set of one or more audits planned for a specific period and a specific purpose [4]. It is more than a plan that presents audit topics, frequencies, and timings. It also provides information necessary to assess the performance of the QMS of a company [5], [6]. The decisions relating to the scope, frequency, and timing of internal quality audits allow for an objective self-introspection of audit program management practices. To understand how the audit program is

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managed in a company, it is necessary to examine the determinants of the audit decisions and the impact of the determinants on the company QMS [1], [3], [6]. This study sought to analyze the internal quality audit program planning and implementation determinants in a South African automotive assembly company called company XYZ. The study is significant to company XYZ and comparable companies for two reasons. The determinants studied will reveal whether company XYZ focuses on 1) compliance with ISO 9001 standards, or 2) customers and continuous improvement.

1.1 | Company Profile

Company XYZ, an automotive assembly plant in South Africa, is part of a global network of 31 plants. It has produced vehicles for over 46 years and has implemented and maintained its integrated quality and environmental and safety management system certifications for over 20 years. Company XYZ is certified to ISO 9001: 2015. However, it is not certified by the automotive QMS, IATF 16949, like other OEMs in the sector.

A study by Msibi and Chiromo at company XYZ found that an increase in the number of internal quality audits conducted according to ISO 19011:2018 guidelines did not correspond to a reduction in nonconformities [8]. Moreover, there was no clarity on the frequency of internal quality audits. This study aimed to analyze the determinants of the company's internal quality audit planning and implementation programs.

2 | Literature Review

ISO 9001 QMS internal quality auditing standard was seriously discussed in the early 1990s [9]. Internal quality audits are conducted for several reasons: to determine whether a QMS conforms to ISO 9001, detect and eliminate nonconformities and their causes, and identify improvement opportunities [10]. Some auditing objectives are to gain certification of the QMS and to ensure the continuity and improvement of the system [11]. Audits shift organizations from reactive to proactive. Organizations seek to address customer complaints, non-conformities, scrap, and rework in a reactive state. Alternatively, proactive organizations seek to improve customer satisfaction; identify opportunities for improvements in product or service, timeliness, and cost reductions [12]. Most studies in the past on internal quality auditing focused on improving the implementation and effectiveness of internal quality auditing practices [9], [12]–[14]. Few papers have been written on internal quality auditing in the automotive industry. Refer to [5], [15], and none discusses internal determinants in the automotive sector. In response to this gap, an empirical study was conducted on the planning and implementation determinants of the QMS audit program in company XYZ.

2.1 | Audit Program Management

To become certified to the ISO 9001:2015 standard, organizations must undergo an initial certification audit and then implement internal quality audit programs [17]. The programs facilitate the conduct of 1st party audits (internal audits), 2nd party audits (external audits) by major customers, and 3rd party audits (external audits) by certification bodies [17].

The typical steps for conducting both internal and external audits are shown in *Fig. 1*. *Step 1* establishes effective communication between the audit team and the auditee. *Step 2* involves deciding the functions and processes to be audited, identifying the physical locations of what is to be audited, and establishing the dates and duration for audit activities.

The fieldwork in *Step 3* involves organizing an opening meeting, the audit interviews, and the closing meeting. Next, the audit report is prepared and distributed in *Step 4* before closing the audit in *Step 5*. Lastly, *Step 6* validates the completion and effect of the actions promised by the auditee [17]. Then the company goes through similar steps when effecting continuous improvement. The migration from one step to another is smooth when there is a unit of purpose between the auditors and the auditees [17].

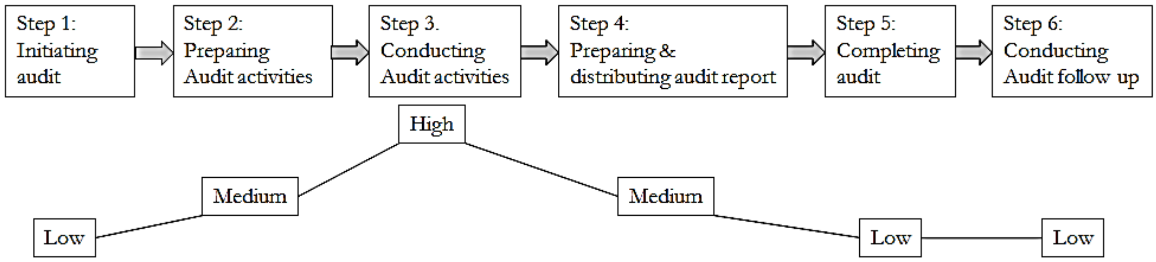

Fig. 1. The steps for conducting the audit process [17].

The ISO 19011:2018 standard provides guidelines for developing and managing audit programs (planning, implementing, monitoring, reviewing, and improving).

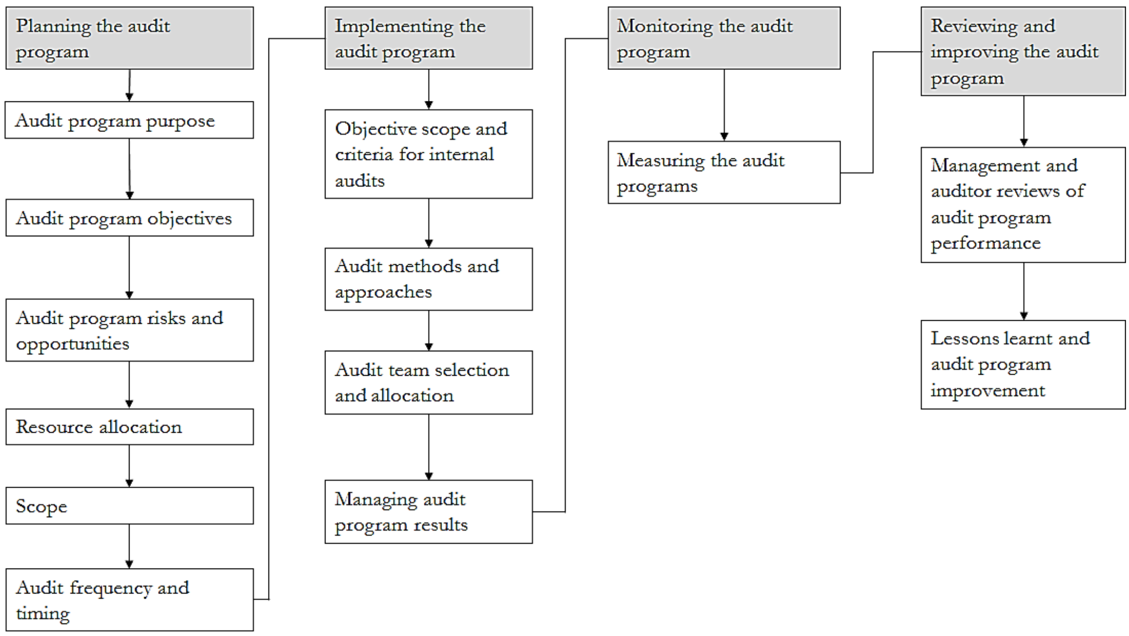

Fig. 2. Internal quality audit program management [6].

In this study, the determinants for internal quality audits for the planning and implementation phases were analyzed in line with the ISO 19011:2018 standards framework (*Fig. 2*).

2.1.1 | Audit program management

An audit program's purpose and objectives are established in the planning phase of an audit program. This phase involves analyzing risks and opportunities; establishing the responsibility for maintaining the audit program; identifying audit resources; and establishing audit scope, frequency, and timing [7], [18]. In addition, the audit program includes planning the opening and closing meetings, reporting audit findings, and making follow-ups on corrective actions [9]. The outcome of the planning phase of the audit program is actionable.

2.1.2 | Implementation of the annual audit program

The implementation of the audits follows the planning phase. This includes defining the objectives and scope of the individual audits, the selection of auditors, and the audit approaches. The effects of audits are a basis for assessing the audit's contribution to achieving business goals and improving the company's

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efficiency [10]. The effects depend on company management's understanding of audits, attitude toward audits, and how management responds to audit findings [10].

2.1.3 | Monitoring of audit program

Monitoring the audit program entails assessing the performance of the audit program [6]. The assessments check the efficiency, effectiveness, and gap between actual and expected performance [1]. Although both efficiency and effectiveness are equally important, effectiveness is commonly used to measure and monitor audit performance [1]. The key measures for measuring and monitoring the effectiveness of an audit program include the number of completed planned audits, the number of identified nonconformities, the number of repeat audit findings, the time taken to issue audit reports, the time taken to implement corrective actions, and the number of process improvements emanating from audit findings [3], [11].

2.1.4 | Review and improvement of the audit program

In reviewing the audit program, top management and individuals responsible for managing the audit program assess whether the objectives of the audit program have been achieved. The lessons from the reviews are then utilized to effect the audit program's improvements [6].

2.2 | Audit Program Planning Determinants

The first step in developing an internal quality program is identifying the determinants for managing the audit program. These determinants cut across the planning, implementation, monitoring, and review phases, including the purpose and objectives, risks and opportunities, audit resources, audit frequency, and timing decisions [3], [5], [7], [18].

2.2.1 | Audit program purpose and objectives

In addition to providing information on failures and conformities, an audit program provides a basis for effecting improvements [3], particularly in response to the call for a systems approach to managing a quality auditing program [18], [19]. The systems approach to managing an audit program views each internal audit within the program as a sub-system of the ISO 9001 QMS, and each internal audit is linked to other internal audits [19].

This means that the planning and implementation of the audit program must focus on the purpose and performance of the overall QMS. Audit program management draws inputs from management, auditees, external quality auditors, and other stakeholders. Moreover, data and information from other systems in the organization are reviewed. The inputs maintain the company's focus on conformity to requirements, customer needs, and continuous improvement [6], [7].

2.2.2 | Audit program risks and opportunities

The principle of Risk-Based Thinking (RBT) generates awareness of the risks and opportunities that influence a company's ISO 9001 QMS performance [5], [18]. The RBT principle must be incorporated in key audit program planning, scope, frequency, and timing [2], [3].

2.2.3 | Audit program resources

The availability of adequate skills in internal quality auditors and the persons responsible for managing audit programs are important when planning an audit program [15]. Internal quality auditors, knowledgeable of the processes and areas to audit, are important determinants. An auditee or an audited organization is also a factor in an effective audit [9]. The auditees work with auditors in the audit program while the auditors are responsible for completing all work according to ISO 9001:2015 standard [9]. People responsible for the audit program's management analyze the system's performance and prepare reports on the overall program efficiency and effectiveness [21]. In cases where managers are not available, technical experts should be engaged [7]. In addition, financial support for the audit program should be allocated [21].

2.2.4 | Audit program scope

Internal quality audits include a company's processes, activities, products, and systems in the audit program's scope. The scope should be narrowed to require fewer audit personnel, shorter audit time, and fewer corrective actions. The narrowing of the scope avoids the generalization of audit findings [22]. An audit based on processes is recommended because it helps internal quality auditors evaluate how the processes' inputs and outputs are managed to achieve the desired results and improve performance [20].

2.2.5 | Audit timing and frequency

Clause 9 of the ISO 9001:2015 standard requires that internal quality audits are planned and conducted regularly. The ISO 19011 standard provides guidelines without suggesting a specific audit frequency [6], [18]. However, it is recommended that audit frequency be based on previous performance and the criticality of the processes [5], [22]. As audits are dynamic and adaptive, the timing and frequency can gradually be improved with time to bring quality improvements to the audit program [7].

There will be no continuous improvement if time is inadequate to conduct audits. A company that wants to be competitive, innovative, and an industry leader must continuously improve its processes and employees [24]. Repeating audits by the same auditors render them predictable, generating less meaningful insight and potentially leading to audit paralysis [14]. Lastly, all planned actions must be followed up timeously, monitored, and evaluated to ensure that the QMSs are maintained over time [9].

2.3 | Audit Program Implementation Determinants

In the execution phase of the audit program, a determination is necessary on how the audit evidence is sampled, how team members are allocated audit assignments, and how corrective audit actions are implemented [6].

2.3.1 | Audit evidence sampling methods

The sampling can be done using either non-probability or statistical probability techniques. When statistical probability techniques are used, the evidence should be selected randomly and be representative of the population. In determining the sample size, intricacy, capacity, risk, past issues, and quality of available data are all assessed [2], [22].

2.3.2 | Audit team selection

The decision to allocate internal quality auditors to specific audit tasks goes beyond the availability of the auditors. Internal quality auditors who are knowledgeable and competent to audit the processes are crucial. Sometimes, it is necessary to pair technical experts with internal quality auditors for specific processes [4].

2.3.3 | Internal quality audit results

Internal quality audit results that provide conformity information only are inadequate as they do not stimulate auditees to improve the processes [5]. Internal quality audit reports need to include improvement recommendations. These recommendations create opportunities that address the findings and rectify issues when planning and implementing corrective actions [3], [14].

2.4 | The Study Contribution

This study contributes to the debate on internal audits by analyzing the audit planning and implementing program determinants in company XYZ. Though the findings are associated with company XYZ and are not generalized to the automotive sector, they initiate a debate on determinants in the automotive sector and other manufacturing sectors. The debate and discussions have a bearing on improving the quality of internal quality audits. Ultimately, the audited companies will transition from reactive to proactive states, which augurs well for companies that seek to be competitive.

3 | Methodology

This project was a stand-alone, single case study that was conducted: 1) to describe the key determinants for planning and implementing audit programs, and 2) to describe how determinants impacted the ISO 9001 QMS performance objectives of company XYZ.

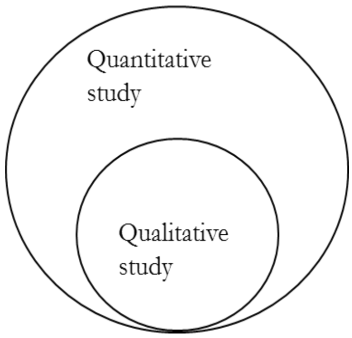

Fig. 3. Qualitative nested in quantitative design [25].

The study was a mixed method, with qualitative nesting data inside quantitative data (*Fig. 3*).

The research provides knowledge practitioners can use at the focal company and other manufacturing companies to improve their internal auditing practices. The conclusions are not generalized beyond the research case.

The literature review revealed the key factors necessary for internal quality audit program planning and implementation.

It also provided knowledge that assisted in analyzing and interpreting secondary data concerning audit program decision determinants [26].

3.1 | Data Collection

The researchers obtained written approval from company XYZ to conduct the study and ethical clearance from the university of Johannesburg. Initially, quantitative data was collected exclusively from historical internal quality audit program records for 2017 to 2020. Then, qualitative data was collected from historical internal quality audit reports. Finally, a retrospective document analysis was conducted to garner insight into the 2017 to 2020 period. This helped the researchers understand how audit program planning and implementation occurred in company XYZ. Qualitative analyses of internal audit reports bolstered the insight gained from analysis of the quantitative data obtained from audit program records.

3.2 | Sampling Technique

A nonprobability judgment sampling technique was used to select the audit reports for analysis. The sample size for the analyzed documents came to 33 audit reports, translating to 20% of the total internal quality reports compiled between 2017 and 2020. This exceeds the minimum sample size (10%) recommended by Mamaile [27]. Having a bigger sample size made it easy to reach the data saturation point for each audit program determinant [26].

Purposive sampling was used to select the interview participants. The chosen people had experience in drawing up and reviewing internal quality auditing programs at company XYZ. They comprised managers, auditors, and auditees.

3.3 | Data Analysis

Descriptive data analysis was adopted to analyze the quantitative data, and deductions were made-thematic analysis of qualitative data elaborated on the understanding gained from the quantitative data. The two data sets provided an integral assessment of the planning and implementation determinants of the internal quality audit program at company XYZ.

4 | Findings

Company XYZ has six quality operation departments; logistics, body shop, paint shop, vehicle assembly, total vehicle quality, and purchasing. Each department's internal quality audit program is managed by a departmental general manager, line manager, and internal quality auditors. The mandate of each team is to manage product quality, process quality, and techniques and tools used to achieve quality compliance. For example, product audits pay attention to the features of a car as it progresses through the production line. On the other hand, process audits pay attention to work instructions and work procedures. Each audit goes through phases comparable to those in *Fig. 1*. Relevant audit checklists, interview techniques, and other tools and techniques are adopted during the audits. Systems audits are conducted at the group level and by personnel outside the assembly plant but are internal at the group level.

The auditors, auditees, and management know the audited products, processes, and systems. The auditees work with auditors during the audit process, and the auditors conduct the audits according to ISO 9001 requirements and ISO 19011: 2018 guidelines. They communicate critical and significant findings to the auditees and the management. All post-audit actions planned are followed-up timeously and monitored and evaluated to ensure that the QMSs are maintained over time.

4.1 | Key Audit Program Planning Determinants

Results from the audit program planning reports revealed that process instability, process non-adherence, and external audit results influenced audit program planning decisions relating to audit objectives, audit scope, and audit frequency. These are discussed in the following sub-sections.

4.1.1 | Audit program objectives determinants

In 2018 and 2019, the audit program objectives sought to meet production requirements and improve quality metrics. In 2020, two additional audit program objectives were added: 1) to support the company's preparedness for external quality audits, and 2) to maintain the ISO 9001 certification. These objectives show the company's desire to enforce process adherence, thereby complying with ISO 9001 QMS requirements. Adopting these objectives reduced the total number of nonconformities from 129 to 20 from 2019 to 2020. Company XYZ regained its QMS certification when external auditors conducted a certification audit in 2019.

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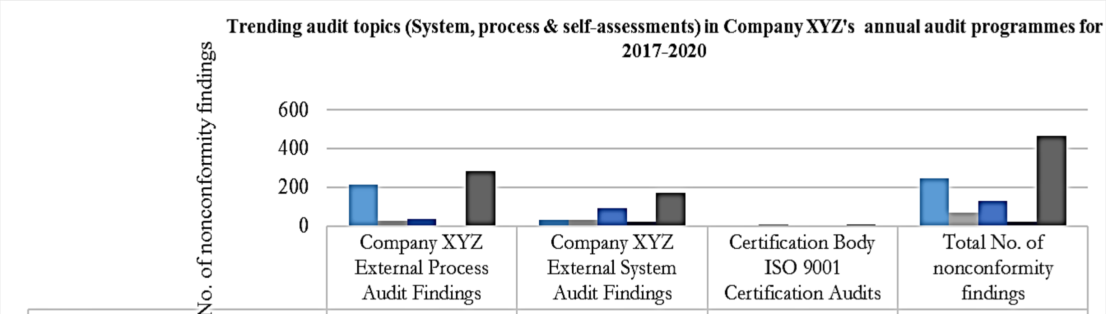

	Company XYZ External Process Audit Findings	Company XYZ External System Audit Findings	Certification Body ISO 9001 Certification Audits	Total No. of nonconformity findings
2017	215	30	0	245
2018	28	32	10	70
2019	38	91	0	129
2020		20	0	20
Accumulated total no. Of external noncormity findings	281	173	10	464

Fig. 4. Distribution of external quality audit nonconformities (2017-2020).

4.1.2 | Audit program scope determinants

The external quality audit results in *Fig. 4* were the main determinants in deciding the scope of the future internal audit programs for processes. The internal quality audits were conducted to identify and correct nonconformities before and after external quality audits. The processes and system internal audits were conducted to assess compliance with ISO 9001 procedures, standards, and guidelines. There is no indication that the audits had a thrust on continuous improvement.

Fig. 5 shows that for the 2017-2020 audit program, 61% of the audits were process audits, 26% were self-assessment audits, 9% were system audits, and 4% were legal compliance and project audits. The statistics in *Fig. 5* show company XYZ's internal quality audit practices prioritized processes [4]. Hoyle argues that process audits extend beyond departmental boundaries and organizational structures, thereby making it possible to assess synergies across organizational units [4]. The transcending of audits across departmental boundaries also renders the call by Alic and Rusjan to adopt a systems approach to internal quality audits significant [13].

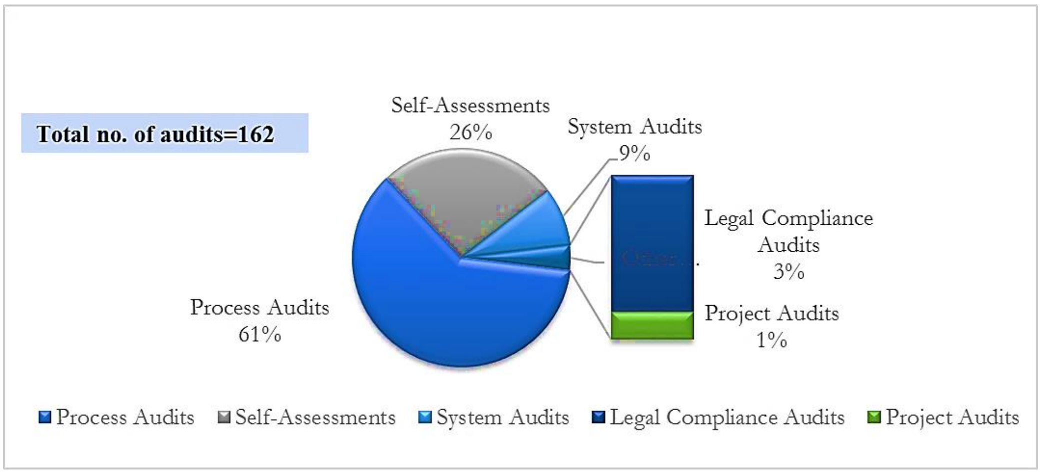

Fig. 5. Company XYZ's audit program distribution (2017-2020).

Table 1 shows that 71 of 104 process audits (68%) at company XYZ evaluated compliance with production procedures and requirements. Combined with audits to prepare the company for certification, the total number of process audits conducted between 2017 and 2022 rose to 87 of 104 (84%).

Self-assessment audits were undertaken to prepare for internal quality audits; this focus constituted 29 of 43 (67%) self-assessment audits conducted between 2017 and 2020, as observed in *Fig. 4*.

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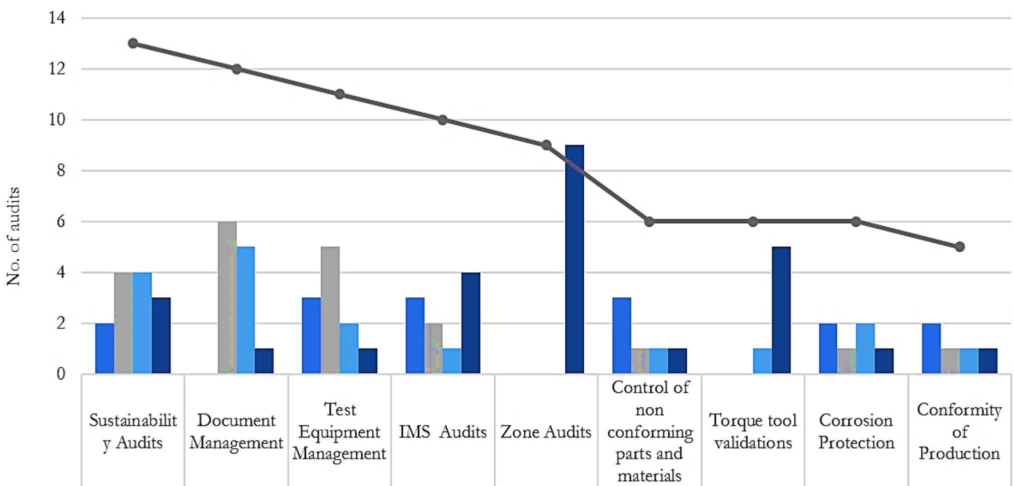
Table 1. The link between audits and audit program objectives at company XYZ.

	Complying with ISO 9001:2018 Requirements	Complying with Production Procedures	Preparation for Certification Audits	Stabilization and Maturing of QMS	Total Audits
Process audits	11	71	16	6	104
System audits	8	-	1	5	14
Self-assessments	-	11	29	3	43
Total number of audits	19	82	46	14	161

Company XYZ planned its audits to identify nonconformities before internal and external quality audits. This allowed the company to maintain its ISO 9001 QMS certification. However, there was little impact on continuous improvement.

The top five audits in company XYZ's annual audit programs in 2017-2020 focused on: 1) sustainability audits, 2) documentation management audits, 3) test equipment management audits, 4) quality, environmental, health, and safety audits, and 5) zone audits (*Fig. 6*). These audits reflect areas from which historic findings emanated. Sustainability and company documents had the highest number of audits between 2017 and 2020. However, auditors were not happy with this arrangement. This could be explained by Lenning and Gremyr [28], who argue that too much focus on documentation and conformity is perceived as a policing activity [28].

Documentation and other internally driven conformity audit also diverted attention from continuous improvement and customer focus objectives. The audits set the company in a reactive rather than a proactive mode.



	Sustainability Audits	Document Management	Test Equipment Management	IMS Audits	Zone Audits	Control of non conforming parts and materials	Torque tool validations	Corrosion Protection	Conformity of Production
2017	2		3	3		3		2	2
2018	4	6	5	2		1		1	1
2019	4	5	2	1		1	1	2	1
2020	3	1	1	4	9	1	5	1	1
Total Audits	13	12	11	10	9	6	6	6	5

Fig. 6. Trending audit topics in Company XYZ's annual audit programs (2017-2020).

4.1.3 | Audit program frequency determinants

The audits in *Fig. 6,* conducted annually by the same auditors, were prompted by external quality audit non-conformities. Deductions from these observations and the best practice audits in the literature show that repeating conformity-driven audit programs for an extended period will lead to audit paralysis. Audit results will be predictable and will cease to reveal meaningful and beneficial insights [14].

4.2 | Key Audit Program Implementation Determinants

Internal audit reports at company XYZ showed that results from past audits, auditees' self-assessments, auditors' judgments, technical experts' knowledge, and document reviews influenced what will be audited. The company's audit programs were still reactive and sought to address previously identified non-conformities. The neglected: improving customer satisfaction; identifying future requirements; and identifying opportunities for improvements in product or service, timeliness, and cost reduction [12].

4.2.1 | Internal quality audit sampling methods

After identifying what is to be audited, the internal auditors of company XYZ relied on their judgment and the advice of technical experts to sample processes, areas, and materials to be audited. They also relied on the reviews of previous audit reports. With this arrangement, the auditors again focused on compliance with requirements rather than on continuous improvement. Audit sampling and auditor selection should occur based on data analysis [2], [3] to ensure that audit samples represent the population [2]. However, relying on random samples without the knowledge of the target processes is costly because the auditees continuously perform self-assessments and pre-audit checks before internal quality audits to comply with requirements. Hence, internal audits need to be information driven.

4.2.2 | Management of internal quality audit results

Between 2017 and 2020, 1882 audit findings were recorded in 162 internal quality audits. The audit findings included conformities, non-conformities, and opportunities for improvement. Audit findings related to ISO 9001 clauses included production control, service provision, compliance monitoring, measuring tools and equipment, document control, and competence management.

The distribution of the findings showed that: 51% were minor non-conformities, 31% were improvements, 10% were positive findings, and 8% were major non-conformities (*Fig. 7*).

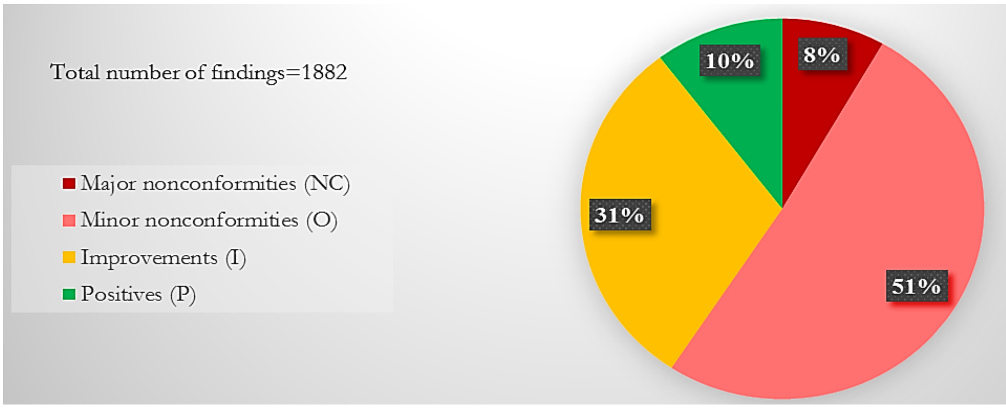


Fig. 7. Distribution of audit findings (NC O, I, and P) at company XYZ (2017-2020).

The sources of nonconformities were production, management reviews, and resource management. There was no clarity on how the auditees identified the root causes of the non-conformities. This led to the auditors rejecting the corrective actions put forward, thereby creating difficulties for the audit program to support the continual improvement objectives of the Company's ISO 9001 QMS.

Table 2. Example of an NC finding with inadequate root causes and corrective actions.

Category of Finding	Area & Implementation Deadlines	Result	Root Causes Actions (N/O only)	Status/Run Time (Days)
NC	Logistics 26/08/2019 30/08/2018	During the audits, information on the competency matrix was not provided.	Competency matrices were incomplete by May 1, 2019. There were no stand-in matrices in new departments.	Verified 67

Sustainability audits were conducted to enforce sustainable corrective actions. An example of a Non-Conformity (NC) finding with inadequate root causes and corrective actions is shown in *Tables 2* and *3* shows examples of NC findings with inadequate explanations provided by the auditors.

Table 3. Examples of NC findings showing inadequate reasons for successful verification.

Category of Finding	Area & Implementation Deadlines	Finding	Root Causes Actions (N/O only)	Proof of Verification
NC	Paint shop 28/07/2018 10/07/2018	The documented process did not conform to the publishing procedure of Company XYZ. The document was marked 'draft' and the owners of the process on the link document were incorrect.	The procedure was unknown.	Action verified 30/07/2018
NC	Body shop 09/06/2017 17/05/2017	The previous monthly destructive testing from the lab only showed in order rating of BI-8. The results from the analysis did not reflect the real condition of vehicles assessed during the audit.	Root cause: I trusted results from the lab, if there were any concerns from them, they would have approached us. Actions: The ratings will be changed for the better. Results will be queried if suspected to be incorrect.	Ok

Company XYZ's adherence to best practices was also questionable, as it downplayed the need for knowledge transfer to influence other areas to improve quality performance. It promoted the idea that the absence of nonconformities, implementing corrective actions, and fulfilling process requirements translated to best practices. Examples of such misleading findings are presented in *Table 4*.

Table 4. Examples of misleading reasons for logging some positive (P) findings.

Category of Finding	Area	Finding
P	Total vehicle quality	The current practice of driving the car after analysis offline poses a risk that the re-worker in the offline area will not capture all defects. The action was: to configure the IPS-Q station in the ATC workstation and also the office workstations in ATC offices with IPS-Q characteristics. This action is still sustained.
P	Assembly	The opening meeting was held with a presentation on what will be looked at during the audit. Target dates were discussed and agreed upon.
P	Purchasing	Audit schedule agreed as well as deadline dates.
P	Assembly	No findings during this audit session.

5 | Discussion

Company XYZ's audit objectives prepared the company for external quality audits; Alic and Rusjan argue that such companies maintain their ISO 9001 QMS to conform to requirements [27]. The company's audit program did not encompass customer focus and continuous improvement.

Company XYZ's excessive focus on conformity objectives underscores underlying problems in the QMS. There is a high likelihood of a lack of adherence to processes in various company departments [1].

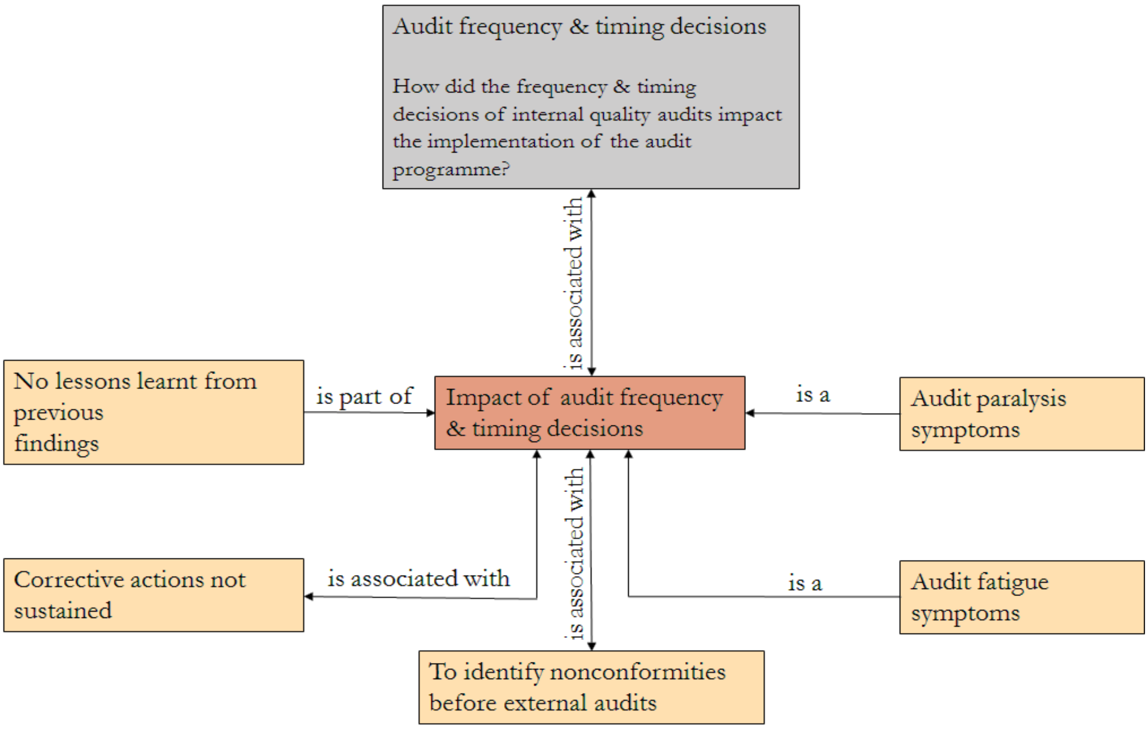


Fig. 8. Internal quality audit frequency and timing decisions (2017-2020).

The nonconformities form the basis for subsequent audit program plans. Company XYZ's focus on conformity objectives could be attributed to its desire to satisfy third-party audits.

Company XYZ's decisions on what is to be audited and on audit frequencies were determined by previous NC audit findings (*Fig. 8*). There are indications that some processes and systems were audited repeatedly while others were rarely. With this approach, processes with fewer or no audit nonconformities are not audited or improved [5]. Insufficient internal quality audits lead to an accrual of nonconformities; eventually, an institution will fail to meet conformity requirements for ISO 9001 QMS certification. On the other hand, excessive auditing leads to audit fatigue and excess auditing time. Too many internal quality audits also deny the company time to implement, control, and improve production processes and quality systems. This eventually leads to paralysis of the audit program.

Another issue that frustrates the internal audit program at company XYZ is the recurrence of audit findings. Nonconformities are not solved once and for all, and corrective actions are not implemented comprehensively. It appears that auditees do not learn from their experiences, and internal auditors repeatedly audit the same areas and processes whenever they conduct audits.

6 | Conclusion and Further Research

At a strategic level, the audits were segmented as system audits, process audits, and product audits. At an operational level, the auditors focused on 1) sustainability, 2) document management, 3) control of production conformity, 4) tools and test-equipment management, and 5) company documentation control.

These policies were developed to; stabilize production and quality metrics, support the company's preparedness for external quality audits, and maintain ISO 9001 compliance certificates. The audits did not adequately address continuous improvement and customer focus, however. Management encouraged the internal auditors to develop audit plans that responded to the outcomes of the previous external audits. Internal quality audits were preoccupied with enforcing conformity to ISO 9001 QMS standards. Consequently, the QMS at Company XYZ remains in a reactive stage and is yet to become proactive.

Recurring nonconformities and failure to attend to root causes hindered implementing sustainable corrective actions. The auditors relied on the personal judgment in sampling processes, products, and systems to be audited. Probability sampling techniques would give management a more astute bird's eye view of the overall performance of the audit process. Moreover, management would better understand how the ISO 9001 QMS is embraced throughout the company.

To elicit a full view of the determinants, further research should analyze all four segments of the audit program: planning, implementation, monitoring, reviewing, and improvement. This will give a more thorough view of the determinants. A better understanding of internal quality audit programs in automotive OEMs would be obtained if a sector-wide study was undertaken. This study's findings are restricted to audit program planning and implementation determinants at company XYZ.

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