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Maximum Number of Parts that can be Accurately Counted Using a Weighing Scale

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Abstract

Many factors may influence the accuracy of part count by weight, but one of the most ubiquitous and often overlooked causes is part variability. In this work, experiments were performed to quantify weight variation of injection molded parts and to measure the maximum number of those parts that could be accurately counted by weight. A model and working equations that account for tolerances of both the mold cavity and plastic were derived to estimate how part variability affects weight counting of a single set of parts. Within experimental uncertainty, the model gave estimates that agreed with the actual part counts.

Keywords: Counting errors, Weight counting, Scale counting, Balance counting.

1 | Introduction

One of the most basic unit operations in manufacturing is counting of parts. In many cases, it is simple. For example, if large or high value items are being produced, those may be packaged one per container. Here, counting is trivial. However, as the number of parts per package rises into the dozens, hundreds or thousands, part counting can become surprisingly complex.

Low mix, high volume operations lend themselves to automation that employs tools such as light counters, vision systems and auto-baggers. While efficient in these settings, fully automated counting is often application specific and requires expensive equipment as well as significant effort to design and implement. On the other hand, high mix, low volume operations usually require flexible counting that can be quickly configured and deployed. Asking an operator to hand count

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large numbers of parts is inefficient and prone to error. Therefore, in these settings, when larger numbers of parts need to be counted quickly, it is often done by weight using scales or balances. As compared to hand counting, scale counting saves time and money. It also can improve accuracy.

Weight counting can be done manually with standard scales or semi-automatically with counting scales. Either way, a sample from a set of parts is weighed to determine an average weight (m), then the full set of parts is placed on the scale to determine its weight (M). The number of parts in the set (n_s) is estimated as:

$$n_s = M/m. \quad (1)$$

The consequences of sending customers the wrong number of parts extend well beyond the cost of missing (or extra) parts. For the customers, too few parts can delay production and create inefficiencies. Too many parts create waste, but also burden customers' quality systems. This is especially true for critical applications such as aerospace and life science where someone's life may depend on assuring that a component or system is properly assembled with the correct number of parts. Also, international shipments that pass through Customs require exact counts of parts. For the manufacturers, the costs also are high. Re-running a few parts to fulfill a short order or properly addressing logistics and quality issues associated with miscounts can amount to thousands of dollars per issue.

One of the challenges that arises in accurately counting by weight is that there always exists some variability in part weight. While using counting scales is quite effective for larger parts, as parts become smaller and more numerous, variation in part weight can lead to counting errors [1, 2]. Even though this is a seemingly important and commonplace issue for manufacturers, surprisingly little has been published on the topic. We briefly summarize two previous papers. Steiner and MacKay (2004) appear to be the first to consider the accuracy of part counting. They performed a statistical analysis of how the variability of individual parts, the number of parts being counted and calibration procedures affect accuracy of part counting [1]. Lee Ho and Cymrot (2008) also analyzed the probabilistic nature of part counting. They derived equations for estimating the confidence intervals associated with counting a given number of parts. Additionally, they performed simulations to understand how many parts must be weighed in the calibration step to attain the desired counting precision [2].

For this paper, experiments were performed to quantify the number of injection molded parts that can be accurately weight counted in our production environment. A simple model was developed to understand how the variability of part weight and part dimensions affects weight count accuracy. Measured values of the part weight variation along with stated tolerances were used to estimate the maximum number of parts that can be accurately counted by weight.

2 | Theory

In this section, the working equations used in our analysis are introduced. Details regarding their derivation can be found in the Appendix.

Maximum number of parts in a single set that can be counted accurately:

Generally, one can count accurately by weight if the sum of the absolute value of the variation in a single set of parts is less than the average weight of an individual part (m). If it is assumed that roughly 95% of the variation in part weight can be approximated as two standard deviations ($2m_\sigma$) [3], then the maximum number of parts that can be counted accurately by weight (nx) can be estimated as,

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$$n_x \approx \frac{1}{2}\frac{m}{m_\sigma}. \quad (2)$$

Estimating weight variation from part volume and tolerances

Parts usually are not designed and manufactured according to nominal weights and acceptable weight variations. Rather they are designed to have nominal dimensions and associated dimensional tolerances. For modern digital manufacturing, it is now commonplace for an injection molded part to be designed with CAD software that yields a 3D model with a nominal volume (V_0). The designer may specify the desired tolerances of the final part or the manufacturer may state the attainable tolerances for the mold and the plastic.

Tolerances represent maximum and minimum dimensional limits. They deviate positively (larger) and/or negatively (smaller) from a nominal value. Tolerances of a mold ($\pm t_c$) arise from deviations that occur during machining of the mold cavity. Plastics are injected into the mold cavity as a molten liquid, then cool to a form a solid. All plastics shrink during cooling. Tolerances of a given plastic ($\pm t_p$) are largely due to variations that occur as it shrinks during cooling and solidification.

Assuming a part can be approximated as a cube and its density is constant throughout, the maximum relative weight variation from nominal ($\Delta m_{\pm,max}/m_0$), where the full extent of the available tolerances are exploited, can be estimated in terms of nominal part volume (V_0) and the absolute values of the mold (t_c) and plastic (t_p) tolerances,

$$\frac{\Delta m_{\pm,max}}{m_0} \approx \pm 3(t_c V_0^{-1/3} + t_p). \quad (3)$$

Eq. (2) provides an estimate of the relative deviation from nominal if the full positive or negative tolerance range is consumed. However, it will be useful to understand the weight variation if only a partial fraction ($\Delta m_{\pm,f}/m_0$) of the tolerances of the mold (f_c) or the plastic (f_p) is expended,

$$\frac{\Delta m_{\pm,f}}{m_0} \approx \pm 3(f_c t_c V_0^{-1/3} + f_p t_p). \quad (4)$$

Where f_c and f_p are both assumed to be < 1.

Estimating maximum accurate weight count from part volume and tolerances

If we can successfully employ *Eq. (3)* and f_c and f_p to describe the observed experimental variation in part weight (m_s/m), then the maximum number of parts that can be counted accurately by weight (n_x) can be estimated from part volume and tolerances,

$$n_x \approx \frac{1}{6}(f_c t_c V_0^{-1/3} + f_p t_p)^{-1}. \quad (5)$$

Our model assumes that only 95% of the variation in part weight ($\pm 2m_s$) is accounted for. Thus, *Eq. (5)* does not assure that part counts will be 100% accurate in all cases where $n < n_x$. Nevertheless, as shown later, it can be useful in understanding the limitation of scale counting and reducing counting errors.

3 | Materials, Molded Parts, Scales and Experimental Methods

The goal of our experiments was to determine the maximum number of parts that can be accurately counted using a weighing scale. To this end, single sets of parts were collected directly from active injection molding machines and analyzed on-site. The mix of part geometries and materials was dictated by the orders being processed during our study. Parts were handled while wearing clean gloves. The number of parts per set ranged from as few as 50 to more than 1000. They were measured, then returned to work flow. Details regarding the materials, part volumes, scales and experimental methods are given below.

3.1 | Plastic Materials

A variety of structural amorphous thermoplastics were used in the study. They included commercial, injection-molding grades of Poly-Carbonate (PC), Acrylonitrile-Butadiene-Styrene (ABS) copolymers, PC/ABS blends, general purpose Poly-Styrene (PS), High Impact Poly-Styrene (HIPS), Poly-Methyl-Meth-Acrylate (PMMA), Poly-Phenylene Ether/Poly-Styrene blends (PPE/PS), Poly-Ether-Imide (PEI), and polyphenylene sulfone (PPSU). The amorphous thermoplastics used here exhibit similar shrinkage during molding, roughly 0.005 – 0.006 cm/cm. Our stated tolerance for the shrinkage of amorphous plastics is t_p = 0.002 cm/cm. (Other types of thermoplastics, such as partially crystalline ones, generally exhibit more shrink and greater variability.)

3.2 | Part Volumes and Geometries

Nominal volume (V_0) of the parts from their 3D models ranged from 0.02 to 84.0 cm^3 (0.001 to 5.13 in^3). To accommodate the shrinkage that occurs during molding, it is standard process that the cavities of the molds are machined to be slightly larger than the final part. If executed with perfect fidelity, then the molded part should be an exact replica of the 3D model, exhibiting the same nominal dimensions and nominal volume. In practice, high fidelity is attained, but it's never perfect. Thus, we state a tolerance range for our mold cavity dimensions of t_c = ±0.008 cm (±0.003 in).

3.3 | Scales

Three different scales were used in our experiments: a small one with 310 g capacity and 0.001 g resolution (Ohaus Adventurer Pro AV313), a medium one with 3 kg capacity with 0.1 g resolution (Ohaus Ranger Count 3000, Model RC31P3) and a large one with 15 kg capacity with 1 g resolution (GSE, Model 675). The scales were validated at the beginning of the project with NIST certified weights (Rice Lake ASTM Range 200 g Precision Laboratory Metric Set, 1 mg to 200 g), then challenged each day with 2g, 20 g, and 200g weights.

3.4 | Determination of Part Weight and Variability

The small capability 310 g scale was used to determine the variability of single sets of parts. Thirty parts were weighed, each one at a time, then an average part weight (m), standard deviation (m_s) and relative standard deviation (m_s/m) were computed. We also measured part weight and standard deviation with slightly smaller and larger sets of parts. Part weight and standard deviation did not vary significantly from the values obtained from the 30-part sets.

3.5 | Preparing the Scales for Counting

Weight counting was done using all three scales, the smallest parts were weighed on 310 g scale, the medium-sized parts on a 3 kg scale and the largest parts on the 15 kg scale.

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The 310 g scale does not directly provide a part count. Here, counting was done manually by inputting weights measured from a set of parts into an external calculator in the form of an Excel spreadsheet, according to *Eq. (1)*.

The 3 kg and 15 kg scales are capable of directly outputting a part count. To prepare them for counting, each was zeroed, a plastic container was placed on the scale, then the scale was tared. Depending on the size of the part, five to twenty parts were placed in the container on the scale. The 3 kg and 15 kg scales were programmed to directly output a part count. For the larger scales, parts were added to attain a minimum weight of 20 g (0.045 lb) for the 3 kg scale or 100 g (0.225 lb) for the 15 kg scale. The number of parts placed in the container were keyed into the 3 or 15 kg counting scale. The counting scales automatically compute an average part weight and use that value along with total weight of the sample set to estimate the count, eq (1). The count was confirmed to be correct on the scale display. Parts were removed from the scale and it was confirmed that count for the empty tared container returned to zero.

3.6 | Counting Methods

Manual and weight counting experiments were done simultaneously. Parts were added to the container on the scale five at a time. The manual count (n_m) and scale count (n_s) were recorded. A weight count ending in a digit other than 0 or 5 indicated an erroneous count. If $n_m \neq n_s$, then the experimentally measured values of n_m, n_s, and n_x were noted. For example, if $n_m = 145$ and $n_s = 144$ and the last accurate weight count was 140 (where $n_m = n_s$), then the experimental value of n_x was recorded as 140. For small parts that were counted on the 310 g scale, once a miscount was observed, five parts were removed, the count was verified to be correct, then parts were added one at a time until an erroneous count was again observed; values of n_m, n_s, and n_x were recorded.

4 | Results & Discussion

The Results and Discussion section is comprised of three parts. In the first part, results from counting experiments are discussed and compared to estimates. In the second part, the variability of part weight is analyzed in greater detail. In the third part, the maximum number of parts that can be counted accurately is estimated from part weight variability and tolerances.

4.1 | Maximum Number of Parts Counted Accurately by Weight

Results from the various counting experiments are shown in *Tables 1* and *2* for parts molded from amorphous plastics having a wide range of shapes, size, and composition. Each entry represents a single set of parts.

Table 1. Data from counting experiments for parts of various nominal volumes (V_0) and compositions.

V_0 (cm³)	V_0 (in³)	Material	m_s/m (%)	Measured n_m	n_s	Estimated* n_x
0.34	0.021	PC/ABS	0.230	199	199	220
0.56	0.034	PEI	0.083	250	250	600
0.64	0.039	ABS	0.130	340	340	390
0.66	0.04	PC	0.097	250	250	520
0.80	0.049	ABS	0.067	200	200	740
0.84	0.051	ABS	0.188	330	330	270
1.16	0.071	ABS	0.280	50	50	180
1.44	0.088	PC/ABS	0.063	500	500	790
2.08	0.127	PC	0.055	500	500	910
2.15	0.131	HIPS	0.203	500	500	250
2.18	0.133	PC	0.076	30	30	660
2.61	0.159	PC	0.036	600	600	1400
4.46	0.272	PC	0.030	100	100	1600
5.21	0.318	ABS	0.015	200	200	3300
5.57	0.340	PC	0.071	340	340	700
7.05	0.430	PC/ABS	0.074	250	250	680
7.34	0.448	PC	0.015	280	280	3400
7.37	0.450	PPE/PS	0.014	250	250	3700
7.93	0.484	PMMA	0.037	275	275	1400
9.14	0.558	PC/ABS	0.074	500	500	670
11.4	0.696	PC/ABS	0.013	500	500	4000
12.7	0.772	ABS	0.068	500	500	730
17.9	1.09	PC	0.042	396	396	1200
22.0	1.35	PC/ABS	0.053	300	300	950
25.6	1.56	ABS	0.195	250	250	260
26.1	1.60	PPE/PS	0.013	150	150	3900
28.8	1.76	ABS	0.012	270	270	4200
31.9	1.95	PEI	0.010	150	150	4900
49.0	2.99	PC	0.015	100	100	3300
58.9	3.59	PPSU	0.019	125	125	2600
84.0	5.13	ABS	0.013	100	100	3900

*Parts of different nominal volumes (V0) and compositions, including the number of components available, were emptied before observing the counting anomaly. m_σ/m is the relative standard deviation of part weight; n_m is the manual part count, n_s is the scale count, and n_x is the maximum number of parts that can be counted accurately by weight.

*Values of n_x were estimated using *Eq. (2)* and tabulated with two significant figures.

Table 1 lists data from counting experiments where the available number of parts were exhausted before a weight counting anomaly was observed. Or in other words, after counting all the parts in one of these sets, the manual count and weight count were equal ($n_m = n_s$). For these parts that were accurately counted by weight, two things were generally true:

- *The parts were not tiny; they had $V_0 > 0.3$ cm³ (>0.02 in³)*
- *These parts exhibited minimal relative variation; all of these had $m_s/m < 0.3\%$; most had $m_s/m < 0.1\%$.*

Also, note that in all but two cases, *Eq. (2)* was correct, *i.e.*, $n_x > n_s$.

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Table 2. Data from counting experiments for parts of various nominal volumes (V_0) and compositions.

V_0 (cm³)	V_0 (in³)	Material	m_σ/m (%)	Measured n_m	n_s	n_x	Estimated* n_x
0.02	0.001	PC	2.03	18	17	17	25
0.02	0.001	PC	2.03	20	19	19	25
0.02	0.001	PC	2.03	20	19	19	25
0.05	0.003	ABS	1.14	27	26	26	44
0.05	0.003	ABS	1.14	24	23	23	44
0.05	0.003	ABS	1.14	25	24	24	44
0.05	0.003	ABS	1.35	54	53	53	37
0.05	0.003	ABS	1.35	54	53	53	37
0.05	0.003	ABS	1.35	53	52	52	37
0.08	0.005	PC	0.792	68	67	67	63
0.18	0.011	PC/ABS	0.389	145	144	140	120
0.80	0.049	PC	0.277	205	204	200	180
0.92	0.056	PMMA	0.148	405	404	400	340
1.23	0.075	PC	0.072	1010	1011	1005	690
2.25	0.137	PC/ABS	0.164	350	351	345	300
18.4	1.12	ABS	0.280	190	191	185	180

*Parts of different nominal volumes (V_0) and compounds include weight counting anomalies.; m_σ/m is the relative standard deviation of part weight; n_m is the manual part count, n_s is the scale count, and n_x is the maximum number of parts that can be counted accurately by weight.

*Values of n_x were estimated using *Eq. (2)* and tabulated with two significant figures.

Table 2 lists data from counting experiments for parts of various nominal volumes (V_0) and compositions where a weight counting anomaly was observed. For the single sets of parts listed in *Table 2*, weight counting failed when the scale no longer indicated the same number of parts as the manual count ($n_m \neq n_s$). When weight counting failed, the following was generally true:

- *The parts were relatively small; they usually had $V_0 < 3$ cm3 (< 0.2 in3)*
- *These parts exhibited greater relative variation; most had $m_s/m > 0.3\%$ and roughly half had $m_s/m > 1\%$.*
- *Strong correlation existed between maximum number of parts that could be accurately weight counted and relative weight variability.*
- *With greater relative variation in part weight, fewer parts could be weight counted accurately.*

Also, the estimated values of the maximum number of parts that could be accurately weight counted (n_x) from *Eq. (2)* agreed reasonably well with the experimental counts. While not precise, the estimated values of n_x were usually within ±30% of the measured maximum weight count.

4.2 | Variability of Part Weight

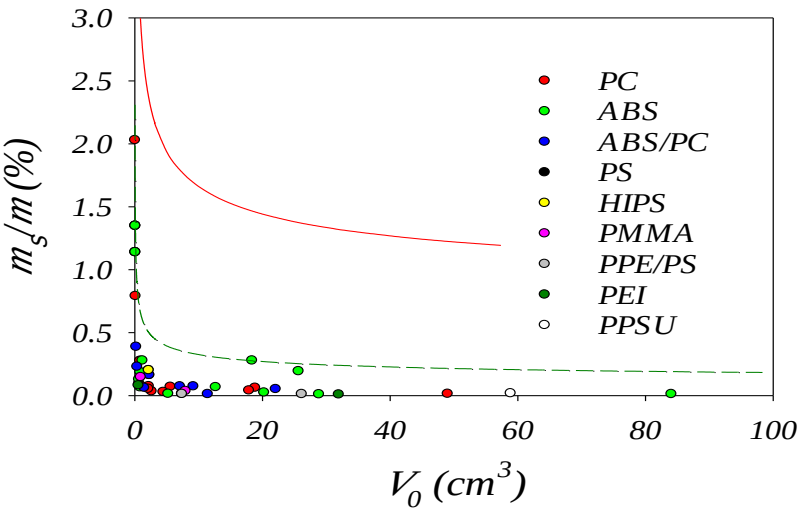

Figure 1. The relative standard deviation of the part weight of all single sets of parts m_σ/m versus part volume (V_0).

In *Fig. 1*, the points represent experimentally measured data. The solid red curve shows an estimate of the absolute value of the maximum relative weight variation from nominal if the tolerances were fully consumed by part variability, according to *Eq. (3)*, where tolerances of the mold and amorphous plastic are t_c = 0.008 cm and t_p = 0.002 cm/cm. The dashed green curve shows an estimate of the absolute value of the relative weight variation if the tolerances were partially consumed by part variability, from *Eq. (4)*, where t_c = 0.008 cm, t_p = 0.002 cm/cm, f_c = 0.25 and f_p = 0.1.

Fig. 1 shows the relative standard deviation m_s/m versus part volume (V_0). Each point represents experimentally measured data from a single set of 30 parts. Plotting the data in this manner reveals that aside from a few outliers, the variability of the part weight varies in a smooth and continuous fashion. For parts larger than 3 cm^3, nearly all had m_s/m < 0.1%. For parts, that were smaller than 3 cm^3, m_s/m increased asymptotically, exceeding 2% for the smallest parts. Neither composition nor geometry appeared to significantly influence the weight variation.

There were two outliers in the vicinity of V_0 = 20 cm^3. The exceptionally large variability of these two parts was attributed to poor design. Both parts had excessively thick walls and lacked fillets and proper coring. Consequently, these parts exhibited flow lines, sink marks and greater weight variation. Other sources of variation could include parting line flash and gate trim vestiges.

The solid red curve in *Fig. 1* shows an estimate of the absolute value of the maximum relative weight variation from nominal if the tolerances were fully consumed by part variability, according to *Eq. (3)*, where tolerances of the mold and amorphous plastic are t_c = 0.008 cm and t_p = 0.002 cm/cm, per our values for amorphous thermoplastics.

The red curve has the same shape as the experimentally measured data, but greatly over predicts their values. Each set of parts was run on a single press over the course of several hours to one day. As a result, the observed weight variation is much less than the variation expected from the full extent of our stated tolerances.

The dashed green line shows an estimate of the absolute value of the relative weight variation, according to *Eq. (4)* with f_c = 0.25 and f_p = 0.1. By assuming that only small fraction of the stated tolerance variation is consumed, the predictive curve drops closer to the measured values. In fitting the data, there was no

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attempt to attain a "perfect" least-squares fit that intersects the points, that would have left roughly half above the predictive curve and the other half below. Rather, the fractional fitting parameters (f_c and f_p) were chosen to provide a buffer such that nearly all points representing the relative variability fell below the predictive curve. This is a conservative approach, as this type of fit captures more of the observed part variability.

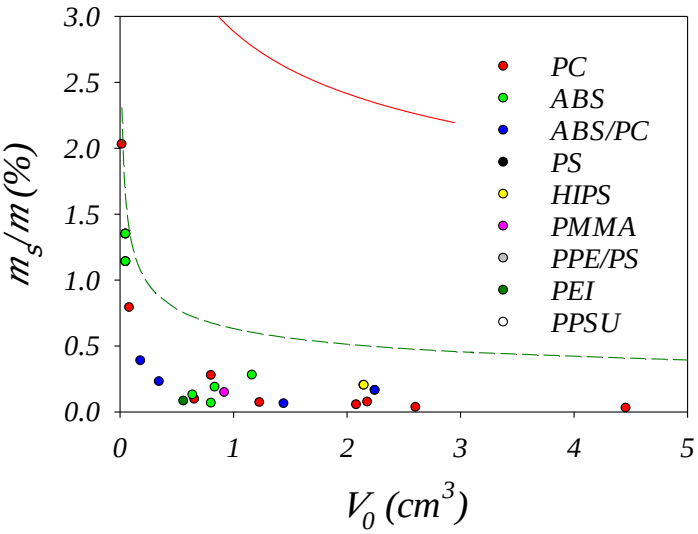

Fig. 2. An alternative representation of data from Figure 1 showing m_σ/m versus volume (V_0) for smaller parts.

In *Fig. 2*, the points are experimental data. The solid red curve shows an estimate of variation from nominal if the tolerances were fully consumed, *Eq. (3)*, $t_c = 0.008$ cm and $t_p = 0.002$ cm/cm. The dashed green curve shows relative weight variation if the tolerances were partially consumed, eq (4), $t_c = 0.008$ cm, $t_p = 0.002$ cm/cm, $f_c = 0.25$ and $f_p = 0.1$.

Fig. 2 is an alternative representation of *Fig.* 1, providing an expanded view of the data for smaller parts where $V_0 < 5$ cm³. Again, the points are experimental data. The red and green curves demonstrate predicted variation from nominal per *Eqs. (3)* and *(4)*. This view provides better visibility near $V_0 = 0$ cm³. As $V_0 \rightarrow 0$ cm³, m_s/m rises asymptotically in a smooth and consistent manner. Also, this view confirms that beyond $V_0 > 3$ cm³, the relative weight variation of these molded parts is indeed small and invariant.

4.3 | Estimating the Maximum Accurate Part Count

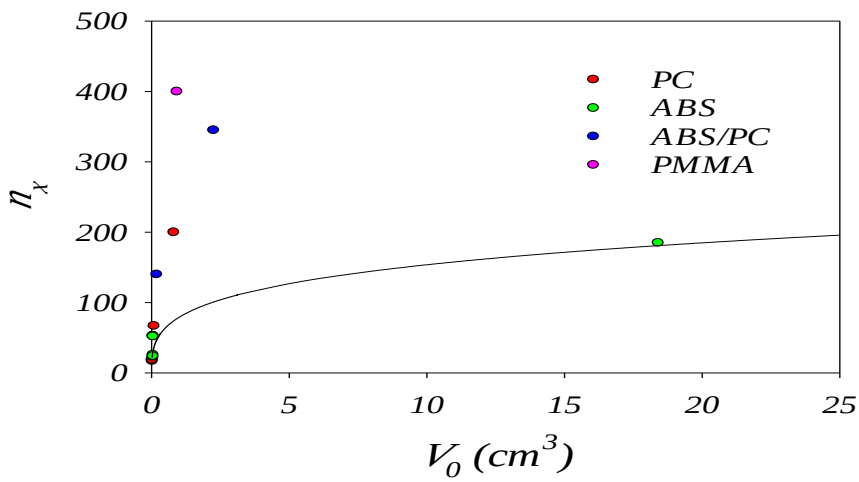

Fig. 3. The maximum number of parts that can be counted accurately by weight (n_x) versus part volume (V_0).

In *Fig. 3*, he points represent experimental data. The curve shows an estimate of n_x from *Eq. (5)*, where t_c = 0.008 cm, t_p = 0.002 cm/cm, f_c = 0.25 and f_p = 0.1.

Fig. 3 shows the maximum number of parts that can be counted accurately by weight (n_x) versus part volume (V_0). The points represent experimental data pulled from *Table 2*. As previously discussed, miscounts were most prevalent for the small parts. With one exception, all the points between n_x values of 0 and 500 resided where $V_0 < 3$ cm^3. Even though the data for relative standard deviation of the part weight manifests as a smooth and continuous curve, this data is probabilistic in nature and thus exhibit scatter, which is easier to see when plotted as n_x against V_0. Parts with less weight variation yielded larger maximum accurate counts, whereas parts with greater variation yielded smaller n_x values.

The curve in *Fig. 3* is an estimate of n_x from *Eq. (5)*, where t_c = 0.008 cm, t_p = 0.002 cm/cm, f_c = 0.25 and f_p = 0.1. If the predicted value of n_x were used as an upper limit for weight counting, points above the curve represent single sets of parts that, in principle, would have been counted a*/ccurately. On the other hand, any points that fell below the curve represent a probable miscount. The model appears to have merit. Nearly all experimental points fell above the predictive curve. This can be attributed in part to the conservative approach used to analyze the experimental data in *Fig. 1* and *2*. The buffered fit employed here captures more of the observed part variability and in turn reduced the probability of a counting anomaly.

5 | Conclusions

Experimental data gathered here supports the main supposition of this work, that variability in part weight can lead to erroneous part counts. It was shown experimentally that the number of parts that can be counted accurately by weight is highly correlated to part weight variability. Part weight variation was relatively small for larger parts, but increased asymptotically as part volume tended towards zero. Also, it was demonstrated that the relative standard deviation in part weight combined with tolerances can be used to make rough estimates of the maximum number of parts that can be accurately weight counted. The work described here focused on amorphous plastics, but the general approach could be applied to other classes of materials.

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We would like to thank our team members: Removed

Declaration of Interest Statement

No financial interest or benefit has arisen from the direct applications of our research.

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Appendix

Derivation of Working Equations

The working equations used earlier for quantifying and understanding weight counting are derived below. For injection-molded parts from a given production order, the variability in part weight is not expected to be normally distributed. The deviation from nominal dimensions of the mold cavity will be disproportionately skewed larger or smaller. In most cases, the deviation of the plastic shrinkage also will be skewed from its nominal value.

Maximum Number of Parts in a Single Set That Can Be Counted Accurately

For our purposes, we define the weight variation of an individual part as the difference between its actual and nominal weights (Δm_i). Generally, one can count accurately by weight if the sum of the absolute values of the weight variation of all parts $\Sigma|\Delta m_i|$ is less than the average weight of an individual part (m),

$$\Sigma|\Delta m_i|<m. \tag{6}$$

The chance of a counting error increases and becomes highly probable if

$$\Sigma|\Delta m_i|>m. \tag{7}$$

At the crossover point where these two quantities are equal, the sum of the absolute values of the weight variation of all parts is denoted as $|\Delta m_i|_x$,

$$\Sigma|\Delta m_i|_x=m. \tag{8}$$

The sum of the absolute values of the weight variation of all parts is equal to the product of the number of parts being counted (n) and the average weight variation of the entire set of parts ($\Delta m_{\pm}$),

$$\Sigma|\Delta m_i|=n\cdot\Delta m_{\pm}. \tag{9}$$

At the crossover point, *Eq. (9)* takes the following form,

$$\Sigma|\Delta m_i|_x=n_x\cdot\Delta m_{\pm}. \tag{10}$$

Where n_x is the maximum number of parts that can be counted accurately by weight. Combining *Eqs. (8)* and *(10)* yields,

$$n_x=\left|\frac{m}{\Delta m_{\pm}}\right|. \tag{11}$$

If it is assumed that the variation in part weight can be approximated as two standard deviations of the part weight ($\pm 2m_s$), capturing roughly 95% of the weight variability [3],

$$\Delta m_{\pm} \approx \pm 2m_{\sigma}. \tag{12}$$

Then *Eqs. (11)* and *(12)* can be combined to arrive at a simple expression that allows estimation of maximum number of parts that can be counted accurately by weight from experimentally measurable quantities, average part weight and the standard deviation,

$$n_x \approx \frac{1}{2}\frac{m}{m_{\sigma}}. \tag{2}$$

Estimating weight variation from part volume and tolerances

Parts usually are designed to have nominal dimensions and associated tolerances. In the case where the full extent of the available tolerances are exploited, the maximum differential weight variation from nominal ($\Delta m_{\pm,max}$) can be estimated for simple geometries in terms the nominal part weight (m_0) along with the upper and lower weight tolerance limits ($m_{\pm,max}$). For the maximum positive deviation from nominal ($\Delta m_{+,max}$), the relative variation is

$$\frac{\Delta m_{+,max}}{m_0} = \frac{m_{+,max} - m_0}{m_0} = \frac{m_{+,max}}{m_0} - 1, \tag{13}$$

Assuming density (ρ) throughout the part is constant, *Eq. (13)* can be recast in terms of volumes,

$$\frac{\Delta m_{+,max}}{m_0} = \frac{\rho \cdot V_{+,max}}{\rho \cdot V_0} - 1 = \frac{V_{+,max}}{V_0} - 1, \tag{14}$$

Where $V_{+,max}$ is the part volume at the upper limit of the weight tolerance. Furthermore, if it is assumed that our parts can be approximated as a cube, then the nominal part dimension (L_0) is directly related to its nominal volume (V_0),

$$L_0 = V_0^{1/3}. \tag{15}$$

The maximum positive deviation of the part volume (V_+) is related to the maximum positive part length ($L_{+,max}$),

$$L_0 = V_0^{1/3}. \tag{16}$$

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The value of $L_{+,max}$ can be determined from the nominal dimension of the cube and the tolerances of the mold (t_c) and plastic (t_p),

$$L_{+,max}=t_c+L_0(1+t_p). \tag{17}$$

Combining *Eqs. (14) - (17)* yields,

$$\frac{\Delta m_{+,max}}{m_0}=\left[t_c V_0^{-1/3}+(1+t_p)\right]^3-1. \tag{18}$$

By expanding *Eq. (18)*,

$$\frac{\Delta m_{+,max}}{m_0}=3t_p+3t_p^2+t_p^3+t_c^3V_0^{-1}+3t_c^2V_0^{-2/3}+3t_c^2t_pV_0^{-2/3}+3t_cV_0^{-1/3}+6t_ct_pV_0^{-1/3}+3t_ct_p^2V_0^{-1/3}, \tag{19}$$

And then eliminating higher order factors (t_p^2, t_p^3, t_c^3, *etc.*) whose values are << t_c or t_p, we arrive at an expression for estimating the maximum positive deviation from nominal ($\Delta m_{+,max}/m_0$) in terms of the mold and plastic tolerances,

$$\frac{\Delta m_{+,max}}{m_0}\approx 3(t_cV_0^{-1/3}+t_p). \tag{20}$$

We found that the difference between the more exact *Eq. (18)* and the approximate *Eq. (20)* is 3% or less for practical values of V_0, t_c and t_p.

A similar expression can be derived for maximum negative deviation from nominal ($\Delta m_{-,max}/m_0$),

$$\frac{\Delta m_{-,max}}{m_0}\approx -3(t_cV_0^{-1/3}+t_p). \tag{21}$$

Thus, by combining *Eqs. (20)* and *(21)*, the maximum weight variation ($\Delta m_{\pm,max}/m_0$), positive or negative, can be written as a single expression,

$$\frac{\Delta m_{\pm,max}}{m_0}\approx \pm 3(t_cV_0^{-1/3}+t_p). \tag{3}$$

Eq. (3) provides an estimate of the relative deviation from nominal if the full positive or negative tolerance range is consumed. However, it will be useful for us to understand the weight variation if only a partial fraction ($\Delta m_{\pm,f}/m_0$) of the tolerances of the mold (f_c) or the plastic (f_p) is expended,

$$\frac{\Delta m_{\pm,f}}{m_0} \approx \pm 3(f_c t_c V_0^{-1/3} + f_p t_p). \quad (4)$$

Estimating maximum accurate weight count from part volume and tolerances

If we can successfully employ *Eq. (4)* and f_c and f_p to describe the observed experimental variation in part weight (m_s/m), then it can be inferred that experimental and predicted part weight variability are approximately equal,

$$\frac{m_\sigma}{m} \approx \left|\frac{\Delta m_{\pm,f}}{m_0}\right|. \quad (22)$$

Subsequently, *Eqs. (2)*, *(4)*, and *(22)* can be combined to yield an expression that enables estimation of maximum number of parts that can be counted accurately by weight from part volume and tolerances,

$$n_x \approx \frac{1}{6}(f_c t_c V_0^{-1/3} + f_p t_p)^{-1}. \quad (5)$$

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